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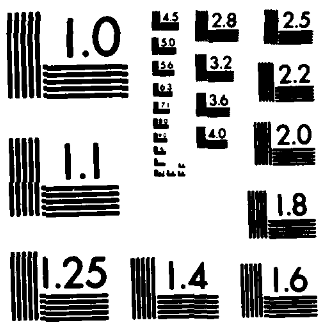


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# FOREIGN TECHNOLOGY DIVISION

EVALUATION OF PARAMETERS OF A SIGNAL

(Selected Pages)

by

S. Ye. Fal'kovich







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(Selected Pages)

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## U. S. BOARD ON GEOGRAPHIC NAMES TRANSLITERATION SYSTEM

| Block | Italic | Transliteration | Block | Italic | Transliteration |
|---|---|---|---|---|---|
| А а | *А а* | A, a | Р р | *Р р* | R, r |
| Б б | *Б б* | B, b | С с | *С с* | S, s |
| В в | *В в* | V, v | Т т | *Т т* | T, t |
| Г г | *Г г* | G, g | У у | *У у* | U, u |
| Д д | *Д д* | D, d | Ф ф | *Ф ф* | F, f |
| Е е | *Е е* | Ye, ye; E, e* | Х х | *Х х* | Kh, kh |
| Ж ж | *Ж ж* | Zh, zh | Ц ц | *Ц ц* | Ts, ts |
| З з | *З з* | Z, z | Ч ч | *Ч ч* | Ch, ch |
| И и | *И и* | I, i | Ш ш | *Ш ш* | Sh, sh |
| Й й | *Й й* | Y, y | Щ щ | *Щ щ* | Shch, shch |
| К к | *К к* | K, k | Ъ ъ | *Ъ ъ* | " |
| Л л | *Л л* | L, l | Ы ы | *Ы ы* | Y, y |
| М м | *М м* | M, m | Ь ь | *Ь ь* | ' |
| Н н | *Н н* | N, n | Э э | *Э э* | E, e |
| О о | *О о* | O, o | Ю ю | *Ю ю* | Yu, yu |
| П п | *П п* | P, p | Я я | *Я я* | Ya, ya |

*ye initially, after vowels, and after ъ, ь; e elsewhere.
When written as ё in Russian, transliterate as yё or ё.

## RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

| Russian | English | Russian | English | Russian | English |
|---|---|---|---|---|---|
| sin | sin | sh | sinh | arc sh | $\sinh^{-1}$ |
| cos | cos | ch | cosh | arc ch | $\cosh^{-1}$ |
| tg | tan | th | tanh | arc th | $\tanh^{-1}$ |
| ctg | cot | cth | coth | arc cth | $\coth^{-1}$ |
| sec | sec | sch | sech | arc sch | $\mathrm{sech}^{-1}$ |
| cosec | csc | csch | csch | arc csch | $\mathrm{csch}^{-1}$ |

| Russian | English |
|---|---|
| rot | curl |
| lg | log |

### GRAPHICS DISCLAIMER

All figures, graphics, tables, equations, etc. merged into this translation were extracted from the best quality copy available.

# EVALUATION OF PARAMETERS OF A SIGNAL.

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## 2. The fundamental principles of the theory of the optimum methods of radio reception.

### 2.1. Statistics of space of oscillations adopted.

In preceding chapter procedure of determination of optimum receiving system and order of calculation of theoretically maximum qualitative indices were in general terms established/installed. Now let us switch over to the investigation of the communication systems with the concrete/specific/actual interferences operating in the radio channels. For this first of all should be described the statistics of the oscillations/vibrations adopted and derived the fundamental principles of the theory of the optimal methods of reception, which include the the analytical expressions of the functions of plausibility and some, connected with them, characteristics for the most widely used models of the communication systems.

Oscillation u(t), which enters into input of radio receiving equipment, in general case is formed/shaped as a result of effect on transmitted signal $s(t; \vec{\lambda})$ of additive and multiplicative

interferences it can be represented in the form

$$u(t) = [\varepsilon_0 + \varepsilon_c(t)]\, s(t;\vec{\lambda}) + \varepsilon_s(t)\, s_\perp(t;\vec{\lambda}) + n(t) =$$
$$= \mathrm{Re}\,[\varepsilon(t)\, e^{j\varphi(t)}\, \dot{s}(t;\vec{\lambda})] + n(t). \qquad (2.1)$$

where $s_\perp(t;\vec{\lambda})$ - conjugated/combined process, connected with transmitted signal s(t; $\vec{\lambda}$) by conversion of Gilbert/Hilbert [24, 45].

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For the usually utilized narrow-band signals adjoint function $s_\perp(t;\vec{\lambda})$ is obtained from function s(t; $\vec{\lambda}$) by phase displacement of the carrier frequency to the angle $\pi/2$; s(t; $\vec{\lambda}$) - complex representation of the signal

$$\dot{s}(t;\vec{\lambda}) = s(t;\vec{\lambda}) + js_\perp(t;\vec{\lambda}). \qquad (2.2)$$

Function n(t) presents additive interferences. To them, first of all, refer the internally-produced noise of equipment, converted to the input. Furthermore, the additive interferences include thermal and space noise received by antenna, different interfering signals as, for example, active jamming. In the radar to the additive interferences, furthermore, relate passive jamming, i.e., reflection from the marine and earth's surface, etc. Additive interferences are usually normal random process with the zero mathematical expectation and and by known correlation function $R_n(t_1;\ t_2)$. Internally-produced noise and some other forms of interferences can be approximated by

the white noises

$$R_n(t_1;t_2)=\frac{N_0}{2}\delta(t_1-t_2), \qquad (2.3)$$

where $N_0/2$ - spectral jamming intensity. Let us note that we use the representation of the frequency spectra on the entire axis - $\infty<f<\infty$. Therefore, spectral intensity figuring in our formulas is 2 times lower than the real spectral intensity $N_0$, by which is understood the power, which falls on 1 Hz of the frequency band.

Idealization of (2.3) in class of tasks considered/examined below is permitted, when interference spectrum is substantially wider than spectrum of signal, and in frequency band, virtually occupied with signal, spectral jamming intensity can be considered constant value. The effect of additive interferences is reduced to the creation of the noise background, against which is realized the reception of useful signals.

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Functions $\varepsilon_0+\varepsilon_c(t)$ and $\varepsilon_s(t)$, and also $\varepsilon(e)$ and $\varphi(t)$:

$$\varepsilon(t)=\sqrt{[\varepsilon_0+\varepsilon_c(t)]^2+\varepsilon_s^2(t)},$$
$$\varphi(t)=\operatorname{arctg}\frac{\varepsilon_s(t)}{\varepsilon_0+\varepsilon_c(t)}, \qquad (2.4)$$

reflect the effect of multiplicative interferences, which appear with passage of signal along channels with variable (in particular, with those fluctuating) parameters. The fluctuations of the parameters of

channel can be caused by different reasons. They include: the multiple-pronged radiowave propagation between the transmitting and receiving points, which leads to the fluctuations of the mutual phasing of the interfering waves, the presence of fluctuating heterogeneities along the path of the radiowave propagation (in the ionosphere and the troposphere), fluctuation of the complex coefficient of reflection of targets (in the radar), fluctuation of the orientation of onboard antenna systems in the space radio communication, etc. As a result of the effect of multiplicative interferences the useful signal undergoes parasitic amplitude and phase modulation. Just as additive interferences, multiplicative interferences are the result of a large number of independent random events, and functions $\varepsilon_c(t)$ and $\varepsilon_s(t)$, which determine their effect, can be assumed/set by the independent normal processes

$$\langle \varepsilon_c(t_1)\varepsilon_s(t_2)\rangle = 0 \qquad (2.5)$$

with the zero mathematical expectation

$$\langle \varepsilon_c(t)\rangle = \langle \varepsilon_s(t)\rangle = 0 \qquad (2.6)$$

and with the identical correlation functions $R_s(t_1;t_2)$:

$$R_s(t_1;t_2) = \langle \varepsilon_c(t_1)\varepsilon_c(t_2)\rangle = \langle \varepsilon_s(t_1)\varepsilon_s(t_2)\rangle. \qquad (2.7)$$

Here and subsequently, brackets of form <> indicate statistical averaging. In the absence of regular component of signal ($\epsilon_0=0$) its phase $\varphi$ and intensity $\epsilon$ at the arbitrary moment of time are distributed evenly to:

$$p(\varphi)=\begin{cases}\frac{1}{2\pi} & \overset{(1)}{\text{при}}\ 0<\varphi<2\pi,\\ 0 & \overset{(1)}{\text{при}}\ \varphi<0 \overset{(2)}{\text{и}}\ \varphi>2\pi,\end{cases} \quad (2.8)$$

Key: (1). with. (2). and.

and according to the law of Rayleigh:

$$p(\varepsilon)=\begin{cases}\frac{2\varepsilon}{\sigma_\varepsilon^2}\exp\left(-\frac{\varepsilon^2}{\sigma_\varepsilon^2}\right) & \overset{(1)}{\text{при}}\ \varepsilon>0,\\ 0 & \overset{(1)}{\text{при}}\ \varepsilon<0,\end{cases} \quad (2.9)$$

Key: (1). with.

where

$$\sigma_\varepsilon^2=\langle\varepsilon_c^2\rangle=\langle\varepsilon_s^2\rangle. \quad (2.10)$$

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Fluctuations of form (2.8)-(2.9) are called Rayleigh. In the general case $\varepsilon_0 \neq 0$ and the intensity of signal $\varepsilon$ is distributed according to generalized Rayleigh's law (to Rice's law)

$$p(\varepsilon)=\begin{cases}\frac{2\varepsilon}{\sigma_\varepsilon^2}\exp\left(-\frac{\varepsilon^2+\varepsilon_0^2}{\sigma_\varepsilon^2}\right)I_0\left(\frac{2\varepsilon\varepsilon_0}{\sigma_\varepsilon^2}\right) & \overset{(1)}{\text{при}}\ \varepsilon>0,\\ 0 & \overset{(1)}{\text{при}}\ \varepsilon<0\end{cases} \quad (2.11)$$

Key: (1). with.

(I - Bessel function from the pure imaginary argument), and phase $\varphi$ - according to the sufficiently complicated law, which with sharply

pronounced regular component ($\varepsilon_0 \gg \sigma_n$) asymptotically approaches the normal:

$$p(\varphi)=\frac{1}{\sqrt{2\pi}\sigma_\varphi}\exp\left[-\frac{(\varphi-\varphi_0)^2}{2\sigma_\varphi^2}\right];$$
$$\varphi_0=\langle\varphi\rangle;\ \sigma_\varphi^2=\langle(\varphi-\varphi_0)^2\rangle. \qquad (2.12)$$

. Thus, in radio engineering communication systems oscillation/vibration u(t) adopted is usually normal random process. For the complete description of this process it suffices to assign the mathematical expectation

$$m(t)=\langle u(t)\rangle=\varepsilon_0 s(t;\vec{\lambda}) \qquad (2.13)$$

and the correlation function $R(t_1;\ t_2)$, which, taking into account the statistical independence of additive and multiplicative interferences, is determined by the expression

$$R(t_1;t_2)=\langle[u(t_1)-\varepsilon_0 s(t_1;\vec{\lambda})]\,[u(t_2)-\varepsilon_0 s(t_2;\vec{\lambda})]\rangle=$$
$$=\mathrm{Re}\,[\dot{s}(t_1;\vec{\lambda})\,\dot{s}^*(t_2;\vec{\lambda})]\,R_\varepsilon(t_1;t_2)+R_n(t_1;t_2). \qquad (2.14)$$

Here and subsequently asterisk with complex quantity indicates complex coupling. Asterisk also designated evaluations/estimates. However, this must not lead to the misunderstandings, since measured parameters and their evaluations/estimates are everywhere real values.

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From (21.3) and (2.14) follows that for the complete statistical

assignment of the oscillations/vibrations (or space U) adopted it is necessary to assign function $s(t; \vec{\lambda})$, which determines the ensemble of the transmitted signals, the intensity of regular component of signal $\varepsilon_0$ and correlation functions $R_n(t_1; t_2)$ and $R_\varepsilon(t_1; t_2)$ of additive and multiplicative interferences.

Mathematical vehicle of statistical theory of connection/communication, based on use of statistical characteristics (2.13) and (2.14), is bulky and is utilized comparatively rarely. We will examine it in Chapter 8. Usually in accordance with the actually occurring working conditions of the majority of the radio engineering communication systems it is examined by one of two idealizations: the case of the slow fluctuations of the parameters of channel (or signal) and the case of the rapid fluctuations of the parameters of channel. The use of these idealizations makes it possible to introduce the assumptions, which significantly simplify mathematical vehicle.

With slow fluctuations of parameters of channel time of correlation of functions $\varepsilon_c(t)$ and $\varepsilon_s(t)$ or $\varepsilon(t)$ and $\varphi(t)$, determining effect multiplicative interferences, is considerably more (at least by an order) interval of observation or duration of signal $s(t; \vec{\lambda})$. In this case it is possible to consider that the functions $\varepsilon(t)$ and $\varphi(t)$ in each interval of observation have, although random, constant

value of $\epsilon$ and $\varphi$. With respect (to 2.1) it is converted in

$$u_s^*(t) = \mathrm{Re}\,[\varepsilon e^{j\varphi} \dot{s}(t;\ \vec{\lambda})] + n(t). \qquad (2.15)$$

Idealized model of channel with rapid fluctuations of parameters is utilized with reception of repeating signals (for example, radar signals), when time of correlation of fluctuations of parameters of channel is less than recurrence interval, but it is considerably more than duration of elementary signals $s_0(t;\ \vec{\lambda})$ in each repetition period. In this case the random processes of $\epsilon(t)$ and $\varphi(t)$ in (2.1) can be replaced with the totalities of statistically independent relative to each other random variables $\vec{\varepsilon} = \varepsilon_1, \ldots, \varepsilon_n;\ \vec{\varphi} = \varphi_1, \ldots, \varphi_n$ and the oscillation/vibration adopted to represent in the form

$$u(t) = \mathrm{Re}\left[\sum_{i=1}^{n} \varepsilon_i e^{j\varphi_i} \dot{s}_0(t - t_i;\ \vec{\lambda})\right] + n(t), \qquad (2.16)$$

where n - number of repetition of signal, and $t_i$ - temporary displacement to the i repetition of the elementary signal $s_0$.

Both idealizations (2.15) and (2.16) are reduced to the fact that effect of multiplicative interferences is considered by addition to transmitted signal of new set of random, but constant parameters $\vec{\alpha}(\vec{\alpha} = \epsilon,\ \varphi$ or $\vec{\alpha} = \vec{\epsilon},\ \varphi)$. Useful signal at the input of the radio receiving device in this case takes the form of the determined function $s(t;\ \vec{\lambda};\ \vec{\alpha})$ of time and two sets of the random parameters $\vec{\lambda}$ and $\vec{\alpha}$, and the oscillation adopted - the additive mixture

$$u(t) = s(t;\ \vec{\lambda};\ \vec{\alpha}) + n(t). \qquad (2.17)$$

Parameters $\vec{\lambda}$, which reflect transmitted communication/report and subjects to evaluation/estimate, are called essential parameters. The parameters $\vec{\alpha}$, which reflect the spurious modulation of signal in the channel, i.e., the effect of multiplicative interferences, to evaluation/estimate are not usually subject and are called the unessential parameters. Let us note that in some systems it nevertheless proves to be advisable to rate/estimate the parameter of intensity of $\epsilon$ or the initial phase $\varphi$.

Thus, in majority of practical interesting cases it is possible to consider that the reception is conducted against the background of additive interferences, and effect of multiplicative interferences to reflect introduction to signal of unessential parameters. The statistics of the space of the oscillations/vibrations adopted in this case is described by the mathematical expectation of the oscillation/vibration

$$m(t) = \langle u(t) \rangle = s(t;\ \vec{\lambda};\ \vec{\alpha}) \qquad (2.18)$$

adopted and by the correlation function

$$R(t_1;\ t_2) = R_n(t_1;\ t_2), \qquad (2.19)$$

which coincides with the correlation function of additive interferences and, therefore, in contrast to (2.14), does not depend

on the transmitted communication/report $\vec{\lambda}$. The latter substantially simplifies investigation.

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In statistical theory of connection/communication also extensively are used idealizations, according to which parameter of intensity of signal $\epsilon$ or parameter of intensity $\epsilon$ and initial phase $\varphi$ are assumed/set by previously known nonrandom values. The models of the communication systems with the signals which do not contain the unessential random parameters (with the known $\epsilon$ and $\varphi$), are called systems with the completely determined signals. Detection and the evaluation of the parameters of the completely determined signals are called simple detection and simple evaluation/estimate. If signal contains the unessential random parameters, then the detection and evaluation of the parameters are called complicated.

In given examination oscillations/vibrations u(t) adopted, and also signals and interference were assumed/set by temporary processes, which enter from antenna input of radio receiver. In this case the task of determining the optimum system does not encompass the synthesis of receiving antenna. The theory, which is based on the examination of time processes, is completely satisfactory when internally-produced noise of equipment are the fundamental source of

interferences. At present ever larger role is begun to play outside interferences. This is caused by the development of space radio links, by the development of the low-noise amplifiers and by the possibility of the effect of electronic jamming. As a result already at the input of receiving antenna electromagnetic field of useful signal is masked with the fields of different interferences, and antenna realizes primary combined working/treatment of signals and interferences. In many instances it respectively proves to be advisable to expand the statistical theory of radio systems and to examine not time signals and interferences (voltage/stress and currents) at the input of receiver, but space-time signals and interferences (electromagnetic field) at the input of the receiving antenna. The results of investigation must determine the optimum system of the space-time processing of signals.

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If the processing can be divided into two consecutive stages - into the three-dimensional and into the time, then optimum antenna and optimum receiver will be determined separately. In the examination of space-time (instead of the temporary/time ones) adopted oscillations/vibrations in all relationships/ratios of present paragraph should be argument t replaced by the vector argument $\vec{\omega}$=t, $\vec{r}$, which is the totality of time t and radius-vector $\vec{r}$ of the points

of real space. The oscillation/vibration adopted will again be the normal random process (now space-time), that exhausts description of which they give the mathematical expectation

$$m(t;\vec{r})=\langle u(t;\vec{r})\rangle \qquad (2.20)$$

and the correlation function

$$R(t_1,t_2;\vec{r}_1,\vec{r}_2)=\langle[u(t_1;\vec{r}_1)-m(t_1;\vec{r}_1)][u(t_2;\vec{r}_2)-$$
$$-m(t_2;\vec{r}_2)]\rangle. \qquad (2.21)$$

Again in general case oscillation/vibration adopted can be described by expression of form (2.1):

$$u(t;\vec{r})=\mathrm{Re}\,[\varepsilon(t;\vec{r})\,e^{j\varphi(t;\vec{r})}\dot{s}(t;\vec{r};\vec{\lambda})]+n(t;\vec{r}), \qquad (2.22)$$

which for majority of in practice interesting cases to admissibly replace with expression, analogous (2.17),

$$u(t;\vec{r})=s(t;\vec{r};\vec{\lambda};\vec{\alpha})+n(t;\vec{r}). \qquad (2.23)$$

2.2. Functional of the probability density of normal random process.

Let us assume, function u(t) is normal random process assigned by mathematical expectation m(t), in general case different from zero, and with correlation function $R(t_1;t_2)$. Is assigned also the interval of observation, i.e., the range of change in argument

$t: t\in(T_1,T_2)$.

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Let us examine first n-dimensional selection $\vec{u}=u_1, \ldots, u_n$, which consists of totality $u_i=u(t_i)$ of the values of process of u(t) at the selective moments of time $t_1, \ldots, t_n$. It is not compulsory, but for future reference is convenient to consider it $t_1, \ldots, t_n$ as such that

$$t_i=T_1+\left(i-\frac{1}{2}\right)\Delta \quad (i=1, \ldots, n), \qquad (2.24)$$

where Δ - interval of discreteness, equal to $T_2-T_1/n$. The examination of selection or digital process $\vec{u}$ is not only the means of approximation/approach to an investigation of continuous process of u(t), but it is also of independent interest, since data about the random process can enter the particular moments of time or process itself it can have discrete/digital character. The totality of values of the mathematical expectation m(t) at the selective moments of time let us designate $\vec{m}=m_1, \ldots, m_n$. With the matrix recording of different expressions of sampling of $\vec{u}$, $\vec{m}$, etc. are considered as column vector. The totality of the values of correlation function at the selective moments of time $R(t_i, t_j)=R_{ij}=R_{ji}$ forms matrix/die R of order n×n, called correlation selection matrix.

Selection or vector $\vec{u}$, is n-dimensional normal random variable, probability density of which is equal to

$$p(\vec{u})=k\exp\left[-\frac{1}{2}\sum_{i,j=1}^{n}\vartheta_{ij}(u_i-m_i)(u_j-m_j)\right], \qquad (2.25)$$

or in matrix recording

$$p(\vec{u})=k\exp\left[-\frac{1}{2}(\vec{u}-\vec{m})^{+}\vartheta(\vec{u}-\vec{m})\right]. \qquad (2.26)$$

Symbol + designated the operation/process of transposition; k - the

constant coefficient, not depending on the concrete/specific/actual realization of selection of $\vec{u}$, or from the vector of the mathematical expectation $\vec{m}$,

$$k=\frac{1}{(2\pi)^{\frac{n}{2}}\sqrt{|R|}}; \qquad (2.27)$$

|R| - determinant of correlation matrix/die $R$; $\vartheta_{ij}$ - matrix elements $\vartheta$, reverse/inverse with respect to the correlation matrix/die R, so that

$$\sum_{j=1}^{n} R_{ij}\vartheta_{jk}=\delta_{ik} \ (\overset{(1)}{\text{или}} \ \vartheta=R^{-1}). \qquad (2.28)$$

Key: (1). or.

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Characteristic of continuous random process u(t), which is continuous analog of multidimensional law of distribution (2.25) and (2.26) with Δ→∞ (or n→∞), is called functional of probability density of process u(t)

$$p\,[u(t)]=\lim_{\substack{\Delta\to 0\\ n\to\infty}} p(\vec{u}). \qquad (2.29)$$

For computing functional p [u(t)] let us introduce function [2, 4]

$$\Theta(t_i;\ t_j)=\lim_{\Delta\to 0}\frac{\vartheta_{ij}}{\Delta^2}, \qquad (2.30)$$

which can be determined from integral equation

$$\int_{T_1}^{T_2} R(t_i;\ t_j)\,\Theta(t_j;\ t_k)\,dt_j=\delta(t_i-t_k), \qquad (2.31)$$

obtaining from equation (2.28), if we latter represent in the form

$$\sum_{j=1}^{n} R_{ij} \frac{\vartheta_{ij}}{\Delta^2} \Delta = \frac{\delta_{ik}}{\Delta}$$

and to pass to limit with Δ→0. Let us note that since according to (2.28) and (2.30) $R_{ij}=R_{ji}$, and $\vartheta_{ij}=\vartheta_{ji}$

$$\theta(t_i;\ t_j)=\theta(t_j;\ t_i). \qquad (2.32)$$

By analogy with the reciprocal matrix ϑ function $\theta(t_i;\ t_j)$ we will call inverse-correlation function, i.e., by the function, reverse/inverse to the correlation.

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The limit of the index of exponential curve (2.25)

$$\lim_{\substack{\Delta\to 0\\ n\to\infty}} \left[-\frac{1}{2}\sum_{i,\ j=1}^{n}(u_i-m_i)\frac{\vartheta_{ij}}{\Delta^2}(u_j-m_j)\Delta^2\right]=$$
$$=-\frac{1}{2}\int_{T_1}^{T_2}\!\!\int [u(t_1)-m(t_1)]\,\theta(t_1;\ t_2)\,[u(t_2)-m(t_2)]\,dt_1dt_2,$$

and the functional of probability density is equal to

$$p[u(t)]=k\exp\left\{-\frac{1}{2}\int_{T_1}^{T_2}\!\!\int [u(t_1)-m(t_1)]\times\right.$$
$$\left.\times\,\theta(t_1;\ t_2)\,[u(t_2)-m(t_2)]\,dt_1dt_2\right\}. \qquad (2.33)$$

Coefficient k, determined by means of (2.27), with n→∞ (Δ→0) is

infinite [2], and, generally speaking, there does not exist final limit for function p [u(t)]. However, since coefficient k does not depend on the realization of process u(t) either on the form of the function m(t), then final limit and physical sense refer of the functionals of probability densities for two realizations of process u(t), let us say u*(t) and u**(t), or for two values of m(t):m*(t) and m**(t). The relation of functionals is shown, how one realization more either is less probable than another, or how more or is less probable one value m(t) in comparison with others during this specific realization of process u(t). Thus, the functional of the probability density of random process u(t) in the general case is determined with an accuracy to coefficient of k. This does not lead to the misunderstandings, since during the solution of practical problems are always examined not functionals themselves, but their relations or logarithmic derivatives, which on coefficient of k do not depend.

Obtained expression (2.33) for functional of probability density of random process u(t) via analogous reasonings can be propagated to some more complicated cases, which are of practical interest. Thus, for instance, instead of the random temporary/time process of u(t) can be examined the random space-time process (field) of $u(t; \vec{r})$.

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In this case scalar argument t should be replaced by vector $\vec{\omega}=t, \vec{r}$. Designating through $\vec{\mathcal{L}}$ the variable range $\vec{r}$ and retaining for t previous range of change, i.e., by accepting $\omega \in \Omega:\{t \in (T_1, T_2); \vec{r} \in \vec{\mathcal{L}}\}$, instead of (2.33) we will obtain

$$p[u(t)] = k \exp\Big\{-\frac{1}{2} \iint_{\vec{\mathcal{L}}} \iint_{T_1}^{T_2} [u(t_1, \vec{r}_1) - m(t_1; \vec{r}_1)]\, \theta(t_1, t_2; \vec{r}_1, \vec{r}_2)\, [u(t_2; \vec{r}_2) - m(t_2, \vec{r}_2)]\, dt_1 dt_2 d\vec{r}_1 d\vec{r}_2\Big\}, \qquad (2.34)$$

where $\theta(t_1, t_2; \vec{r}_1, \vec{r}_2)$ - the function, reverse/inverse of the space-time correlation function $R(t_1, t_2; \vec{r}_1, \vec{r}_2)$:

$$\int_{\vec{\mathcal{L}}} \int_{T_1}^{T_2} R(t_1, t_2; \vec{r}_1, \vec{r}_2)\, \theta(t_2, t_3; \vec{r}_2, \vec{r}_3)\, dt_2 d\vec{r}_2 = \delta(t_1 - t_3)\, \delta(\vec{r}_1 - \vec{r}_3). \qquad (2.35)$$

Subsequently in present chapter all relationships/ratios are given for temporary/time processes. These relationships/ratios if necessary can be, it is analogous how this was done upon transfer from (2.33) to (2.34), they were propagated to the space-time processes by means of the replacement

$$t \overset{(1)}{\text{на}}\ \vec{\omega} = t,\ \vec{r} \overset{(2)}{\text{и}}\ t \in (T_1, T_2) \overset{(1)}{\text{на}}\ \vec{\omega} \in \Omega:\{t \in (T_1, T_2); \vec{r} \in \vec{\mathcal{L}}\}.$$

Key: (1). on. (2). and.

Let us note also that matrix recording (2.26) can in very general view represent as discrete/digital, so also continuous processes. Actually/really, in accordance with what has been said it is higher than the vector $\vec{u}$ and $\vec{m}$ can present continuous processes of u(t) and m(t), and also u(t; $\vec{r}$) and m(t, $\vec{r}$), and matrix - correlation functions and function inverse by correlation ones.

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After the appropriate determination of the scalar and matrix product of functions expressions (2.33) and (2.34) can be converted in (2.26).

In practical problems multidimensional (vector) random processes $\vec{u}(t)$, which are totality of one-dimensional (scalar) random processes $\vec{u}(t)=u_1(t), \ldots, u_n(t)$, in general case correlated with each other, are also encountered. The functional of the probability density of this process

$$p[\vec{u}(t)]=p[u_1(t), \ldots, u_n(t)]= \\ =k\exp\left\{-\frac{1}{2}\sum_{i,\,j=1}^{n}\iint_{T_1}^{T_2}[u_i(t_1)-m_i(t_1)]\,\theta_{ij}(t_1;\,t_2)\times \right. \\ \left. \times[u_j(t_2)-m_j(t_2)]\,dt_1dt_2\right\} \quad (2.36)$$

will be required subsequently. Here, as above, $\Theta_{ij}(t_1;\ t_2)$ - the function, reverse/inverse correlation $R_{ij}(t_1,\ t_2)$, determined from the equation

$$\int_{T_1}^{T_2} R_{ij}(t_1;\ t_2)\,\Theta_{ij}(t_2;\ t_3)\,dt_2 = \delta(t_1 - t_3), \qquad (2.37)$$

and

$$R_{ij}(t_1;\ t_2) = \langle [u_i(t_1) - m_i(t_1)]\,[u_j(t_2) - m_j(t_2)] \rangle \qquad (2.38)$$

and

$$m_i(t) = \langle u_i(t) \rangle. \qquad (2.39)$$

If processes $u_i(t)$ with different values of index i are not correlated, then with i≠j value $R_{ij}(t_1;\ t_2) = 0$ and (2.36) is converted in

$$p[\vec{u}(t)] = k \exp\left\{ -\frac{1}{2} \int_{T_1}^{T_2}\!\!\int [u_i(t_1) - m_i(t_1)]\,\Theta_{ii}(t_1;\ t_2) \times \right.$$
$$\left. \times [u_i(t_2) - m_i(t_2)]\,dt_1 dt_2 \right\}. \qquad (2.40)$$

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## 2.3. Function of plausibility.

In accordance with the definition, given above, function of plausibility, which defines optimum output effect of radio receiving equipment, represents conditional probability density $p(\vec{u}|\vec{\lambda})$ of oscillation/vibration $\vec{u}$, which is considered as function of parameters of signal $\vec{\lambda}$ (transmitted communications/reports) at this recorded realization of oscillation/vibration $\vec{u}$ adopted. The form of the function of plausibility depends on the correlation function of

additive interferences, and also on whether they are considered and how multiplicative interferences, in particular, which of the unessential parameters of signal are taken into consideration. In other words, the form of the function of plausibility depends on what is utilized the model of the oscillations/vibrations $\vec{u}$ adopted.

Let us begin examination from simplest model, according to which oscillation/vibration adopted presents additive mixture of interference n(t) with assigned correlation function $R_n(t_1;\ t_2)$ and completely determined signal $s(t;\ \vec{\lambda})$:

$$u(t)=s(t;\ \vec{\lambda})+n(t). \tag{2.41}$$

Mathematical expectation of normal process (2.41) is equal to $s(t;\ \vec{\lambda})$. Therefore the function of plausibility, i.e., the conditional functional of probability density $\vec{u}$, according to (2.33) takes the form

$$p(\vec{u}\mid\vec{\lambda})=k\exp\left\{-\frac{1}{2}\iint\limits_{T_1}^{T_2}[u(t_1)-s(t_1;\ \vec{\lambda})]\times\right.$$
$$\left.\times\Theta(t_1;\ t_2)[u(t_2)-s(t_2;\ \vec{\lambda})]\,dt_1dt_2\right\}. \tag{2.42}$$

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Function $\Theta(t_1;\ t_2)$, as in all other cases, it is determined from equation (2.31), which in case (2.41) of reception in question against the background of additive interferences is converted in

$$\int_{T_1}^{T_2} R_n(t_1;\ t_2)\,\theta(t_2;\ t_3)\,dt_2 = \delta(t_1 - t_3). \qquad (2.43)$$

Due to limitedness of integration limits in equation (2.43), and also other analogous equations, which determine function Θ, solution in final form succeeds in obtaining hardly ever. Simplest is obtained solution with the widely utilized approximation of additive interferences by white noises (2.3). In this case

$$\theta(t_1;\ t_2) = \frac{2}{N_0}\,\delta(t_1 - t_2) \qquad (2.44)$$

and

$$p(\vec{u}\,|\,\vec{\lambda}) = k\exp\left\{-\frac{1}{N_0}\int_{T_1}^{T_2}[u(t) - s(t;\ \vec{\lambda})]^2\,dt\right\}. \qquad (2.45)$$

Overwhelming majority of practical results of statistical theory of radio installations is connected with use of latter/last relationship/ratio.

More complicated, but is also more acceptably solution of equation (2.43) for stationary correlated interferences, if time of correlation of interferences is considerably (at least by an order) less than duration of interval of observation $T_2$-$T_1$. In this case (2.43) it is converted into the fold of functions $R_n$ and Θ, and solution is obtained by the use/application of Fourier transform:

$$\theta(t_1;\ t_2) = \theta(t_1 - t_2) = \int_{-\infty}^{\infty} e^{j2\pi f(t_1 - t_2)}\,df\left[\int_{-\infty}^{\infty} R_n(t)\,e^{-j2\pi ft}\,dt\right]^{-1}. \qquad (2.46)$$

Let us introduce following designations:

$$q(\vec{\lambda})=\frac{2}{N_0}\int_{T_1}^{T_2}u(t)\,s(t;\ \vec{\lambda})\,dt=$$
$$=\left[\int\int_{T_1}^{T_2}u(t_1)\,\theta(t_1;\ t_2)\,s(t_2;\ \vec{\lambda})\,dt_1dt_2\right] \qquad (2.47)$$

and

$$\mu(\vec{\lambda})=\frac{E(\vec{\lambda})}{N_0}=\left[\frac{1}{2}\int\int_{T_1}^{T_2}s(t_1;\ \vec{\lambda})\,\theta(t_1;\ t_2)\,s(t_2;\ \vec{\lambda})\,dt_1dt_2\right]. \qquad (2.48)$$

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In first part of equalities (2.47) and (2.48) are represented expressions, which relate to approximation of interferences by white noises. To this approximation in accordance with the conventional practice we in essence will be oriented with the following presentation. However, the formulas given below are accurate also for the correlated interferences, if function $q(\vec{\lambda})$ and $\mu(\vec{\lambda})$ are represented by the general/common/total expressions, included in (2.47) and (2.48) into the brackets. Symbol E designates the integral

$$E(\vec{\lambda})=\int_{T_1}^{T_2}s^2(t;\ \vec{\lambda})\,dt, \qquad (2.49)$$

which is proportional to energy and it is respectively called energy of signal. In the general case the energy of signal can depend on the value of some of the random parameters of signal $\lambda_1, \ldots, \lambda_n$. The parameters which affect energy of signal, call the energy parameters,

while the parameters which do not affect energy of signal, by manpower parameters. Value $\mu$ is called energy relation signal/noise. Taking into account the introduced designations of relationship/ratio (2.45) and (2.42) it is possible to replace with one expression

$$p(\vec{u}\,|\,\vec{\lambda}) = k\exp\left[-\mu(\vec{\lambda}) + q(\vec{\lambda})\right], \qquad (2.50)$$

in which coefficient k depends on $\vec{u}$, but not on $\vec{\lambda}$.

According to data of u(t) accepted it suffices to form function $q(\vec{\lambda})$ so that it would be possible to determine function of plausibility (and also if necessary a posteriori distribution) without new turning to u( t). The first and main stage in perfecting of the oscillation/vibration adopted, therefore, consists of the formation/education of function $q(\vec{\lambda})$, called correlation integral.

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This function determines those essential operations/processes, which must be fulfilled above u(t) in order to extract entire available information about the transmitted communication/report. Therefore they indicate that function $q(\vec{\lambda})$ in the general case is a sufficient statistics or the output effect of a sufficient receiver. If all parameters of signal are manpower, which fairly often is assumed/set in the theoretical models of the communication systems, then energy of signal and energy relation signal/noise are constant values:

$E(\vec{\lambda})=E$; $\mu(\vec{\lambda})=\mu$. In this case factor $\exp(-\mu)$ in (2.50) can be connected with constant coefficient of k, and the function of plausibility is one-to-one (exponential) function with respect to the correlation integral $q(\vec{\lambda})$. Respectively in the manpower parameters of signal function $q(\vec{\lambda})$ can be accepted as the output effect of optimum radio receiving equipment.

Let us assume now, that received signal is function of transmitted communication/report $\vec{\lambda}$ and certain set $\vec{\alpha}$ of unessential parameters

$$u(t)=s(t;\ \vec{\lambda};\ \vec{\alpha})+n(t). \qquad (2.51)$$

On the basis of that presented higher it is possible to immediately record function of plausibility $p(\vec{u}|\vec{\lambda};\ \vec{\alpha})$ for entire set $\vec{\lambda}$, $\vec{\alpha}$ of random parameters of signal: both essential and not essential. Function $\vec{p}(u|\vec{\lambda};\ \vec{\alpha})$ can be, for example, represented by expression (2.50), in which only should be replaced the totality $\vec{\lambda}$ by $\vec{\lambda}$, $\vec{\alpha}$. The function of plausibility $\vec{p}(\vec{u}|\vec{\lambda})$ only for the essential parameters of signal, the determining structure of optimum receiving system, is found from function $p(\vec{u}|\vec{\lambda};\ \vec{\alpha})$ with the aid of conversion (2.52), to which they come by means of the following reasonings. Let us record a posteriori probability density $p(\vec{\lambda};\ \vec{\alpha}|\vec{u})$ for the essential and unessential parameters, taking into account, that, as a rule, the unessential parameters $\alpha$ do not contain useful information, i.e., it is statistically independent with respect to $\vec{\lambda}$:

$$p(\vec{\lambda},\ \vec{\alpha}\,|\,\vec{u})=\frac{p(\vec{\lambda})\,p(\vec{\alpha})\,p\,(\vec{u}|\vec{\lambda};\ \vec{\alpha})}{p(\vec{u})}.$$

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A posteriori probability density $p(\vec{\lambda}|\vec{u})$ for essential parameters is only determined by integrating latter/last expression for set A of possible values of unessential parameters $\vec{\alpha}$:

$$p(\vec{\lambda}|\vec{u}) = \int_{A} p(\vec{\lambda};\ \vec{\alpha}|\vec{u})\, d\vec{\alpha} = \frac{p(\vec{\lambda})}{p(\vec{u})} \int_{A} p(\vec{\alpha})\, p(\vec{u}|\vec{\lambda};\ \vec{\alpha})\, d\vec{\alpha}.$$

On the other hand,

$$p(\vec{u}|\vec{\lambda}) = \frac{p(\vec{\lambda})\, p(\vec{u}|\vec{\lambda})}{p(\vec{u})},$$

whence

$$p(\vec{u}|\vec{\lambda}) = \int_{A} p(\vec{\alpha})\, p(\vec{u}|\vec{\lambda};\ \vec{\alpha})\, d\vec{\alpha}. \qquad (2.52)$$

Thus, function of plausibility $p(\vec{u}|\vec{\lambda})$ for essential parameters is only obtained from function of plausibility $p(\vec{u}|\vec{\lambda}, \vec{\alpha})$ for entire set of random parameters of signal via statistical averaging from unessential parameters.

Using obtained law (2.52), let us find function of plausibility for models of signals with most frequently those utilized at theoretical studies by unessential parameters.

Signal with unknown initial phase $\varphi$. Received signal is represented in the form

$$s(t;\ \vec{\lambda};\ \varphi) = \mathrm{Re}\,[\dot{s}(t;\ \vec{\lambda})e^{j\varphi}] = s(t;\ \vec{\lambda})\cos\varphi + s_{\perp}(t;\ \vec{\lambda})\sin\varphi, \tag{2.53}$$

where $s(t;\ \vec{\lambda})$ - as is above, the completely determined signal, i.e., the signal, which represents during each given transmitted communication $\vec{\lambda}$ the known function of time.

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Initial phase $\varphi$ is assumed/set the independent (from $\vec{\lambda}$) random variable, evenly distributed in interval of 0, $2\pi$. In addition to (2.47) and (2.48) let us introduce the designations:

$$q_{\perp}(\vec{\lambda}) = \frac{2}{N_0}\int_{T_1}^{T_2} u(t)\, s_{\perp}(t;\ \vec{\lambda})\, dt = \left[\iint_{T_1}^{T_2} u(t_1)\,\theta(t_1;\ t_2)\, s_{\perp}(t;\ \vec{\lambda})\, dt_1 dt_2\right] \tag{2.54}$$

and

$$Q(\vec{\lambda}) = \sqrt{q^2(\vec{\lambda}) + q_{\perp}^2(\vec{\lambda})}; \tag{2.55}$$

$$\Phi = \mathrm{arctg}\,\frac{q_{\perp}(\vec{\lambda})}{q(\vec{\lambda})}.$$

Here and subsequently, analogous with that as this was accepted in (2.47) and (2.48) into brackets consist expressions, which relate to general case of correlated interferences.

Taking into account, that

$$q(\vec{\lambda})\cos\varphi + q_{\perp}(\vec{\lambda})\sin\varphi = Q(\vec{\lambda})\cos(\Phi+\varphi),$$

we find

$$p(\vec{u}\,|\,\vec{\lambda}) = \frac{1}{2\pi}\int_0^{2\pi} p(\vec{u}\,|\,\vec{\lambda},\ \varphi)\,d\varphi =$$

$$= k\exp[-\mu(\vec{\lambda})]\,\frac{1}{2\pi}\int_0^{2\pi}\exp[Q(\vec{\lambda})\cos(\Phi+\varphi)]\,d\varphi$$

or

$$p(\vec{u}\,|\,\vec{\lambda}) = k\exp[-\mu(\vec{\lambda})]\,I_0[Q(\vec{\lambda})], \qquad (2.56)$$

where $I_0$ - Bessel function from pure imaginary argument.

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Function $Q(\vec{\lambda})$ according to determination of (2.55) is modular value of complex correlation integral $\dot{Q}(\vec{\lambda})$:

$$\dot{Q}(\vec{\lambda}) = q(\vec{\lambda}) + jq_{\perp}(\vec{\lambda}) = \frac{1}{N_0}\int_{T_1}^{T_2} u(t)\,\dot{s}^*(t;\ \vec{\lambda})\,dt =$$

$$= \left[\frac{1}{2}\iint_{T_1}^{T_2}\dot{u}(t_1)\,\Theta(t_1;\ t_2)\,\dot{s}^*(t_2;\ \vec{\lambda})\,dt_1 dt_2\right], \qquad (2.57)$$

where $\dot{u}(t)$ - complex representation of oscillation/vibration u(t) adopted. With the reception of signals with the unknown initial phase function $Q(\vec{\lambda})$ plays the role, analogous to function $q(\vec{\lambda})$ with the reception of the completely determined signals. In particular, function $Q(\vec{\lambda})$ is sufficient statistic and can be accepted as the

output effect of optimum (when $\mu(\vec{\lambda})$=const) or sufficient (when $\mu(\vec{\lambda})$=const) radio receiving equipment.

Signal with unknowns by initial phase and intensity. In this case

$$s(t;\ \vec{\lambda};\ \varphi;\ \epsilon) = \mathrm{Re}\,[\epsilon e^{j\varphi}\dot{s}(t;\ \vec{\lambda})] =$$
$$= \epsilon s(t;\ \vec{\lambda})\cos\varphi + \epsilon s_{\perp}(t;\ \vec{\lambda})\sin\varphi. \qquad (2.58)$$

Let us record the function of plausibility for the entire set of the random parameters of signal $\vec{\lambda}$, $\varphi$, $\epsilon$:

$$p(\vec{u}\,|\,\vec{\lambda},\ \varphi,\ \epsilon) = k\exp[-\epsilon^2\mu(\vec{\lambda}) + \epsilon Q(\vec{\lambda})\cos(\Phi+\varphi)].$$

The standardization of the parameter of intensity $\epsilon$ let us select by such that $\langle\epsilon^2\rangle$=1. In this case $\mu(\lambda)$ presents the mathematical expectation of energy relation the signal/noise:

$$\mu(\vec{\lambda}) = \left\langle \frac{1}{N_0}\int_{T_1}^{T_2} s^2(t;\ \vec{\lambda};\ \varphi;\ \epsilon)\,dt \right\rangle_{\epsilon} =$$
$$= \left[\left\langle \frac{1}{2}\iint_{T_1}^{T_2} s(t_1;\ \vec{\lambda};\ \varphi;\ \epsilon)\,\theta(t_1;\ t_2)\,s(t_2;\ \vec{\lambda};\ \varphi;\ \epsilon)\,dt_1dt_2 \right\rangle_{\epsilon}\right]. \quad (2.59)$$

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Averaging of function $p(\vec{u}|\vec{\lambda},\ \varphi,\ \epsilon)$ on $\varphi$ and $\epsilon$ in the case of Rayleigh fluctuations

$$p(\varphi)=\begin{cases}\frac{1}{2\pi} \text{ при } \varphi\in 0;\ 2\pi,\\ 0 \text{ при } \varphi\bar{\in} 0;\ 2\pi,\end{cases} \qquad (2.60)$$

$$p(\varepsilon)=\begin{cases}2\varepsilon\exp(-\varepsilon^2) & \text{при } \varepsilon\geqslant 0,\\ 0 & \text{при } \varepsilon<0\end{cases}$$

Key: (1). with.

it gives [11, 45, 58, etc.]

$$p(\vec{u}\,|\,\vec{\lambda})=\frac{k}{\mu(\vec{\lambda})+1}\exp\left[\frac{1}{4}\frac{Q^2(\vec{\lambda})}{\mu(\vec{\lambda})+1}\right]. \qquad (2.61)$$

As in the preceding case, as output effect of optimum (or sufficient) receiver can be accepted modular value $Q(\vec{\lambda})$ of complex correlation integral (2.57). This possibility is retained also at other laws of distribution of the parameter of the intensity $\varepsilon$ [49].

Incoherent packet of signals. Signal in this case takes the form of the sum

$$s(t;\ \vec{\lambda};\ \vec{\varepsilon};\ \vec{\varphi})=\sum_{i=1}^{n}\varepsilon_i s_i(t;\ \vec{\lambda};\ \varphi_i)=$$
$$=\sum_{i=1}^{n}\varepsilon_i s_i(t;\ \vec{\lambda})\cos\varphi_i+\varepsilon_i s_{i\perp}(t;\ \vec{\lambda})\sin\varphi_i, \qquad (2.62)$$

of mutually orthogonal signals $s_i$, so that

$$\frac{2}{N_0}\int_{T_1}^{T_2} s_i(t;\ \vec{\lambda};\ \varphi_i)\, s_j(t;\ \vec{\lambda};\ \varphi_j)\, dt =$$

$$= \left[\iint_{T_1}^{T_2} s_i(t_1;\ \vec{\lambda};\ \varphi_i)\,\theta(t_1;\ t_2)\, s_j(t_2;\ \vec{\lambda};\ \varphi_j)\, dt_1 dt_2\right] =$$

$$= \begin{cases} \mu_i(\vec{\lambda}) & \text{при } i = j, \\ 0 & \text{при } i \neq j. \end{cases} \qquad (2.63)$$

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Initial phases $\varphi_i (i=1, \ldots, n)$ are random mutually independent variables, evenly distributed in interval (0; 2π). To the class of signals in question relates, for example, incoherent packet of the repeating signals

$$s_i(t;\ \vec{\lambda};\ \varphi_i) = s_0\,[t - (i-1)\,T_0;\ \vec{\lambda};\ \varphi_i], \qquad (2.64)$$

widely utilized as the model of the echo radar signal. Elementary signals $s_i$ can differ from each other also of carrying by frequency, form, type modulation.

After averaging on basis of random parameters $\varphi_1, \ldots, \varphi_n$ function of plausibility $p(\vec{u}|\vec{\lambda};\ \vec{\epsilon})$ for measured parameters and set of parameters of intensity is equal to

$$p(\vec{u}\,|\,\vec{\lambda};\ \vec{\epsilon}) = k\sum_{i=1}^{n} \exp\,[-\epsilon_i^2 \mu_i(\vec{\lambda})]\; I_0\,[\epsilon_i Q_i(\vec{\lambda})]. \qquad (2.65)$$

Function $Q_i(\vec{\lambda})$ - modular value, complex correlation integral for

elementary signal $s_i$ - is determined by means of (2.57) under condition of replacement s and $s_i$.

Depending on the static characteristics of set of parameters of intensity $\vec{\varepsilon}=\varepsilon_1, \ldots, \varepsilon_n$ three models of incoherent packets of signals are distinguished.

1. Systems with signals of fixed/recorded intensity. In this case

$$\varepsilon_1 = \ldots = \varepsilon_n = 1 \tag{2.66}$$

$$p(\vec{u} \mid \vec{\lambda}) = k \exp\left\{-\mu(\vec{\lambda}) + \sum_{i=1}^{n} \ln I_0 \left[Q_i(\vec{\lambda})\right]\right\}, \tag{2.67}$$

where $\mu_i(\vec{\lambda})$ - complete energy relation signal/noise for the received signal,

$$\mu(\vec{\lambda}) = \mu_1(\vec{\lambda}) + \ldots + \mu_n(\vec{\lambda}). \tag{2.68}$$

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With accuracy, sufficient for practical applications entering into (2.67), the sum is equal to

$$\sum_{i=1}^{n} \ln I_0(Q_i) = \begin{cases} \frac{1}{4}\sum_{i=1}^{n} Q_i^2 \text{ при } Q_i \ll 1 \ (\mu_i \ll 1), \\ \sum_{i=1}^{n} Q_i \text{ при } Q_i \gg 1 \ (\mu_i \gg 1). \end{cases} \quad (2.69)$$

Key: (1). with.

2. Systems with "harmnious" fluctuations of elementary signals. All parameters of intensity are equal to each other $\varepsilon_1 = \ldots = \varepsilon_n = \varepsilon$ and it is random variable with the assigned law of distribution p($\varepsilon$). In this case

$$p(\vec{u}\,|\,\vec{\lambda}) = k \int_0^{\infty} p(\varepsilon) \exp\left\{-\varepsilon^2 \mu(\vec{\lambda}) + \sum_{i=1}^{n} \ln I_0[\varepsilon Q_i(\vec{\lambda})]\right\} d\varepsilon. \quad (2.70)$$

3. Systems with independent fluctuations of elementary signals. All parameters of intensity $\varepsilon_1, \ldots, \varepsilon_n$ are the random mutually independent variables with the identical law of distribution p($\varepsilon_i$):

$$p(\vec{u}\,|\,\vec{\lambda}) = k \prod_{i=1}^{n} \int_0^{\infty} p(\varepsilon_i) \exp[-\varepsilon_i^2 \mu_i(\vec{\lambda})]\, I_0[\varepsilon_i Q_i(\vec{\lambda})]\, d\varepsilon_i. \quad (2.71)$$

Assuming that parameter of intensity is distributed on Rayleigh's law, we find

$$p(\vec{u}\,|\,\vec{\lambda})=\frac{k}{[\mu_1(\vec{\lambda})+1]\cdots[\mu_n(\vec{\lambda})+1]}\times$$
$$\times\exp\left[\frac{1}{4}\sum_{i=1}^{n}\frac{Q_i^2(\vec{\lambda})}{\mu_i(\vec{\lambda})+1}\right]. \qquad (2.72)$$

2.4. Structure of optimum systems.

In preceding paragraph functions of plausibility and optimum output effects of radio receiving equipment with different models of signal were determined.

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Now let us examine in general terms the structure of the optimum diagrams of working/treatment on the assumption that the additive interferences are not correlated, i.e. they are determined by relationship/ratio (2.3). For simplicity of reasonings we will assume that the measured parameter λ is scalar. With the completely determined signal as the optimum output effect of Y(λ) it is possible to accept the correlation integral

$$Y(\lambda)=\int_{-\infty}^{\infty}u(t)\,s(t;\ \lambda)\,dt \qquad (2.73)$$

or any informational equivalent (2.73). Taking into account, that function s(t; λ) out of the integral ($T_1$, $T_2$) is identically equal to zero, integration in the correlation integrals here and subsequently

is propagated to the infinite limits. The reproduction of function $Y(\lambda)$ in entire a priori interval $\lambda \in (\lambda_{мин}, \lambda_{макс})$ technically is difficultly realized. An exception is the case, when the parameter $\lambda$ is the delay time of signal or the parameter, linearly connected with the delay time. In all other cases is reproduced not the continuous function $Y(\lambda)$, but the totality of its discrete/digital values $Y(\lambda_1), \ldots, Y(\lambda_m)$ in interval $\lambda \in (\lambda_{мин}, \lambda_{макс})$. The system of optimum working/treatment in this case is constructed according to multichannel diagram as this shown in Fig. 2.1.

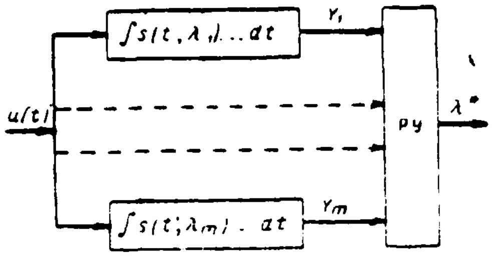


Fig. 2.1. Multichannel diagram of optimum working/treatment.

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In each channel above oscillation adopted is realized linear operation/process - formation of correlation integral (2.73) with fixed value of parameter $\lambda=\lambda_i$ $(i=1, \ldots, m)$. Output effects $Y_i=Y(\lambda_i)$ of all channels are fed/conducted to resolver RU, which makes decision of $\lambda^*$ the maximum of plausibility. M number of the discrete/digital values of the parameter λ or number of independent channels of working/treatment is defined, as this will be shown subsequently, by concrete/specific/actual signal aspect and by length of a priori interval $\lambda_{макс}-\lambda_{мин}$. The simplest and general/common/total consideration is the fact that the selection of discrete/digital values $Y(\lambda_1), \ldots, Y(\lambda_m)$ must with the accuracy, sufficient for virtually optimum making of a decision, reproduce function Y(λ) in the interval of possible values of λ.

Depending on method of formation of correlation integral are distinguished two equivalent methods of optimum reception: correlation method and method of optimum filtration. The block diagram of one channel of reception with the correlation method is shown in Fig. 2.2. Diagram consists of the device/equipment, which generates the multiplication of oscillation accepted by reference oscillation $s(t;\lambda_i)$, which coincides in the form with the expected signal, and of the device/equipment, which realizes integration of product. With the second method of reception (Fig. 2.3) basic element is optimum linear filter OF with the pulse response (i.e. reaction to the input effect $\delta(t)$)

$$h(t) = Cs(t_0 - t;\ \lambda_i), \qquad (2.74)$$

which is the mirror image of signal $s(t;\ \lambda_i)$ relative to axis t=0, shifted to $t_0$. Coefficient $C$ considers the possibility of the selection of arbitrary amplification. The temporary displacement $t_0$ also is to a considerable extent arbitrary.

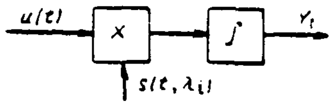


Fig. 2.2.


Fig. 2.3.

Fig. 2.2. Schematic of correlation channel of working/treatment.

Fig. 2.3. Diagram of formation of correlation integral by optimum filtration.

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The sole limitation, which is superimposed on it - this is the condition of the physical realizability of the filter

$$h(t)=0 \text{ with } t<0. \qquad (2.75)$$

If input of optimum filter oscillation u(t) enters, then output effect

$$v(t)=\int_{-\infty}^{\infty} u(x)h(t-x)\,dx=C\int_{-\infty}^{\infty} u(x)s(x+t_0-t;\ \lambda_i)\,dx \qquad (2.76)$$

at moment of time $t=t_0$ is value of correlation integral $Y(\lambda_i)$. When the useful parameter is delay time $s(t;\ \lambda)=s(t-\tau)$, the output effect

of the optimum filter

$$v(t)=C\int_{-\infty}^{\infty} u(x)\,s(t_0+x-t)\,dx \qquad (2.77)$$

in interval $t\in(t_0+\tau_{мин},\ t_0+\tau_{макс})$ reproduces the correlation integral $Y(\tau)$ for all values of time delays interval $\tau\in(\tau_{мин},\ \tau_{макс})$. The method of optimum filtration, therefore, in the case, when the measured parameter is coded in the time delay of signal, provides the formation of continuous output effect with the use instead of the multichannel schematic (Fig. 2.1) of one channel of reception (Fig. 2.3).

In random phase of received signal (and also at to random phase and intensity) as optimum output effect of arbitrary i channel of processing it is possible to accept modular value of complex correlation integral

$$Z(\lambda_i)=\sqrt{\left[\int_{-\infty}^{\infty} u(t)\,s(t;\ \lambda_i;\ \varphi)\,dt\right]^2+\left[\int_{-\infty}^{\infty} u(t)\,s_{\perp}(t;\ \lambda_i;\ \varphi)\,dt\right]^2}, \qquad (2.78)$$

where s(t; λ; φ) - signal, which contains two parameters: measured λ and immeasurable - initial phase φ; $s_{\perp}$ (t; λ; φ)function, conjugated/combined (actually quadrature, i.e. out of phase on 90°) with respect to s(t; λ; φ).

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Correlation method of formation of optimum output effect (2.78)

is represented in Fig. 2.4. Diagram consists of two quadrature channels, which form correlation integrals $Y_i = Y(\lambda_i)$ and $Y_{i\perp} = Y_\perp(\lambda_i)$. As the supporting/reference oscillations are utilized oscillations $s(t; \lambda_i; \varphi)$ and $s_\perp(t; \lambda_i; \varphi)$. Outputs of both channels are fed/conducted to the device/equipment, which realizes an operation/process $\sqrt{Y_i^2 + Y_{i\perp}^2}$.

Fig. 2.5 shows by filter method of formation of modular value of correlation integral, which is reduced to transmission of oscillation u(t) accepted through optimum filter with pulse reaction (2.74), which follows linear amplitude detector D, which realizes operation of obtaining by envelope applied to its input oscillation. Initial phase of pulse response (2.74), just as the supporting/reference oscillation $s(t; \lambda_i; \varphi)$ in the diagram in Fig. 2.4, arbitrary. At the moment of time $t=t_0$ at the output of system (Fig. 2.5) optimum output effect (2.78) is formed/shaped.

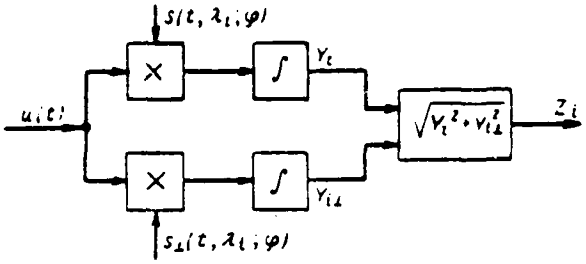


Fig. 2.4.

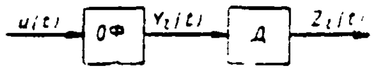


Fig. 2.5.

Fig. 2.4. Schematic of optimum correlation channel of processing signals with random initial phase.

Fig. 2.5. Diagram of processing signals with random initial phase by method of optimum filtration.

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If the measured parameter is coded in the time delay of signal, single-channel diagram in Fig. 2.5 again reproduces the continuous output effect of $Z(\tau)$ on entire multitude of possible values of the measured parameter $\tau$. The need in the multichannel diagram in this case drops off.

With reception of packet of incoherent repeating signals in the

case, when delay time ($\lambda=\tau$) is measured parameter; according to (2.67), (2.69) and (2.72) optimum output effect depending on value of relation signal/noise for elementary signal and on character of fluctuations of intensity can be represented in the form

$$Z(\tau)=\sum_{i=1}^{n} Z_0[\tau-(i-1)T_0] \quad (2.79)$$

or

$$Z(\tau)=\sum_{i=1}^{n} Z_0^2[\tau-(i-1)T_0], \quad (2.80)$$

where $Z_0(\tau)$ - envelope ($l$) of oscillation at output of optimum filter, matched with elementary signal $s_0(t)$:

$$Z_0(\tau)=l\left[\int_{-\infty}^{\infty} u(t)s_0(t-\tau)\,dt\right]. \quad (2.81)$$

Examination of expressions (2.79)-(2.81) makes it possible to conclude that optimum processing of incoherent repeating signals consists of intraperiod and interperiod processing. Intraperiod working/treatment is reduced to the optimum filtration of elementary signals and to the linear (2.79) or quadratic (2.80) detection. In the process of interperiod working/treatment it is conducted the addition (accumulation) of the oscillations, which enter from the output of the detector through the intervals of time $T_0$, equal to repetition period. In the general case the addition can be conducted with certain weight coefficient $\gamma_i=\gamma(iT_0)$. For the realization of this operation/process the summing or storage device/equipment must be linear system with the pulse reaction

$$h_n(t)=\gamma(t)\sum_{i=-\infty}^{\infty}\delta(t-iT_0). \quad (2.82)$$

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Function $\gamma(t)$ plays role of cutting function, which determines effective number of additions. In particular, in (2.79) and (2.80)

$$\gamma(t)=\begin{cases}1 \text{ при}^{(1)}\ t\geq 0 \text{ и}^{(2)}\ t\leqslant nT_0,\\ 0 \text{ при}^{(1)}\ t<0 \text{ и}^{(2)}\ t>nT_0.\end{cases} \quad (2.83)$$

Key: (1). with. (2). and.

Designating Fourier transform ($\mathcal{F}$) of cutting function by symbol $\Gamma(f)$ $(\Gamma(f)=\mathcal{F}[\gamma(t)])$, we find that frequency characteristic of storage system H(f) is equal to

$$H(f)=\mathcal{F}[h_n(t)]=\frac{1}{T_0}\sum_{i=-\infty}^{\infty}\Gamma\left(f-\frac{i}{T_0}\right) \quad (2.84)$$

and has type of rack/comb, depicted (in initial section of positive semi-axis f) in Fig. 2.6. Therefore the diagrams of the accumulation of the repeating signals are called also comb filters. Form and bandwidth of the separate racks/combs of optimum comb filter are determined according to (2.84) by form and by the effective duration of the cutting function $\gamma(t)$. With the rectangular cutting functions (2.83) rack/comb they take the form

$$\frac{\sin \pi nT_0\left(f-\frac{i}{T_0}\right)}{\pi nT_0\left(f-\frac{i}{T_0}\right)}.$$

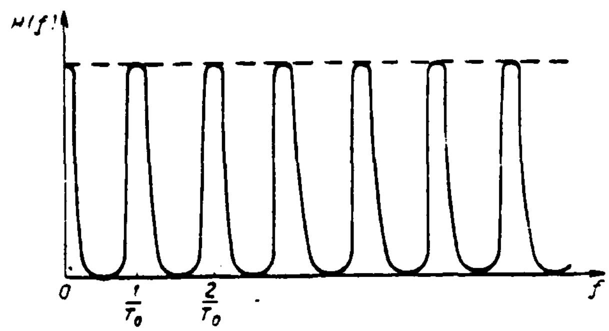


Fig. 2.6. Frequency characteristic of comb filter.

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It is possible not to worry about a precise reproduction of the optimum form of the racks/combs of filter. Essential losses in this case will not be. It suffices to ensure virtually their optimum width, approximately/exemplarily equal to reciprocal value of the effective duration of the cutting function, with the arbitrary form of separate racks/combs.

As example let us give diagram and characteristics of one of simplest comb filters, depicted in Fig. 2.7. Diagram consists of the linear quadrupole K and delay line LZ, connected with regenerative feedback loop. The transmission factors of quadrupole and delay line are equal to k and $\beta$ respectively, and delay time in line $t_з = T_0$. Transmission factor on the ring of feedback $\beta k$ is expressed through

the auxiliary coefficient $\alpha$:

$$\beta k = e^{-\alpha T_0} (\beta k < 1)$$

or

$$\alpha = \frac{1}{T_0} \ln \beta k.$$

Then pulse reaction $h_u(t)$ of the considered/examined system will take the form

$$h_u(t) = k[\delta(t) + e^{-\alpha_0 T}\delta(t - T_0) + e^{-2\alpha T_0}\delta(t - 2T_0) + \dots] =$$
$$= k e^{-\alpha t} \sum_{i=0}^{\infty} \delta(t - iT_0).$$

We respectively find the Fourier transform of the cutting function

$$\Gamma(f) = \int_0^{\infty} k e^{-\alpha t} e^{-j2\pi f t} dt = \frac{k}{\alpha} \frac{1}{1 + j\frac{2\pi f}{\alpha}}$$

and the frequency characteristic of comb filter, substituting latter/last expression in (2.84).

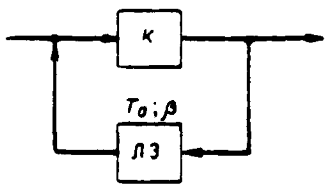


Fig. 2.7. Block diagram of comb filter.

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Separate of rack/comb filter reproduce the frequency characteristic of single resonant circuit with the passband P (at the level -3 dB), equal to $\alpha/\pi$. Consequently, the bandwidth of the separate racks/combs of filter is connected with the transmission factor on the ring of feedback with the relationship/ratio

$$\beta k = \exp(-\pi \Pi T_0).$$

The given examination makes it possible to conclude that optimum filter (rarer, the correlator) is fundamental element of optimum system. The methods of the synthesis of optimum filters for different signals, including for the signals of complex form, are at present developed in sufficient detail [25, 58, etc.]. If it is not possible to approximate additive interference by white noise, the pulse response of the optimum filter

$$h(t) = \sigma(t - t_0;\ \lambda) \qquad (2.85)$$

and reference signal $\sigma(t, \lambda)$ optimum correlator are determined by the

relation

$$\sigma(t;\ \lambda)=\int_{T_1}^{T_2}\theta(t;\ t_1)\,s(t_1;\ \lambda)\,dt_1, \qquad (2.86)$$

which does not change the essence of the task of guaranteeing the optimum reception.

2.5. Signal functions and ambiguity function.

Let us examine form of the function $q(\vec{\lambda})$ and $Q(\vec{\lambda})$, which determine output effect of optimum receiver, for which in more detail let us assume that true value $\vec{\lambda}$ in realization $\vec{u}$ adopted is equal to $\vec{\lambda}_{\text{н}}$.

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Then in the model of system with the completely determined signals

$$u(t)=s(t;\ \vec{\lambda}_{\text{н}})+n(t) \qquad (2.87)$$

and

$$q(\vec{\lambda})=q_s(\vec{\lambda}_{\text{н}};\ \vec{\lambda})+q_n(\vec{\lambda}), \qquad (2.88)$$

where

$$q_s(\vec{\lambda}_u;\ \vec{\lambda}) = \frac{2}{N_0}\int_{-\infty}^{\infty} s(t;\ \vec{\lambda}_u)\ s(t;\ \vec{\lambda})\,dt =$$

$$= \left[\iint_{-\infty}^{\infty} s(t_1;\ \vec{\lambda}_u)\,\Theta(t_1;\ t_2)\,s(t_2;\ \vec{\lambda})\,dt_1dt_2\right]; \qquad (2.89)$$

$$q_n(\vec{\lambda}) = \frac{2}{N_0}\int_{-\infty}^{\infty} n(t)\,s(t;\ \vec{\lambda})\,dt =$$

$$= \left[\iint_{-\infty}^{\infty} n(t_1)\,\Theta(t_1;\ t_2)\,s(t_2;\ \vec{\lambda})\,dt_1dt_2\right]. \qquad (2.90)$$

Function $q_s(\vec{\lambda}_u;\ \vec{\lambda})$ is useful or signal component of output effect, whereas $q_n(\vec{\lambda})$ - noises, which mask signal function $q_s$ at the output and which are normal random process with zero mathematical expectation and with correlation function

$$\langle q_n(\vec{\lambda}_1)\,q_n(\vec{\lambda}_2)\rangle =$$

$$= \frac{4}{N_0^2}\iint_{-\infty}^{\infty} \langle n(t_1)\,n(t_2)\rangle\ s(t_1;\ \vec{\lambda}_1)\,s(t_2;\ \vec{\lambda}_2)\ dt_1dt_2 =$$

$$= \left[\iiiint_{-\infty}^{\infty} \langle n(t_1)\,n(t_2)\rangle\,\Theta(t_1;\ t_3)\,\Theta(t_2;\ t_4)\,s(t_3;\ \vec{\lambda}_1)\,s(t_4;\ \vec{\lambda}_2)\ \times\right.$$

$$\left.\times\ dt_1dt_2dt_3dt_4\right] = q_s(\vec{\lambda}_1;\ \vec{\lambda}_2), \qquad (2.91)$$

of that coinciding in form with signal function $q_s$.

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The ratio of the power of signal component of optimum output effect to the dispersion of interference component at the point of the true

value of the parameter $(\vec{\lambda}=\vec{\lambda}_{u})$

$$\frac{q_s^2(\vec{\lambda}_n;\ \vec{\lambda}_u)}{\langle q_n^2(\vec{\lambda}_n)\rangle}=2\mu(\vec{\lambda}_n) \qquad (2.92)$$

is equal to the doubled energy relation signal/noise at the input of system.

In theoretical models of communication systems essential parameters of signal $\vec{\lambda}$ in majority of cases are such, that signal function is symmetrical function of difference in its arguments

$$q_s(\vec{\lambda}_1;\ \vec{\lambda}_2)=q_s(\vec{\lambda}_1-\vec{\lambda}_2)=q_s(\vec{\lambda}_2-\vec{\lambda}_1) \qquad (2.93)$$

and has maximum (maximum maximorum) in zero $\vec{\lambda}_1-\vec{\lambda}_2=0$, equal to $2\mu$:

$$q_s(0)=2\mu=\text{Max}.$$

Condition of nonenergetics of parameter $\vec{\lambda}$ is necessary, but not sufficient so that signal function would take form (2.93). It is not difficult to show (by the expansion of function $s(t;\ \vec{\lambda})$ in (2.98) in the power series in the vicinity of point $\vec{\lambda}=\vec{\lambda}_n$), that during the transmission of the scalar parameter $\vec{\lambda}=\lambda$ relation (2.93) occurs, if not only energy of signal, but also energy of all its derivatives from the parameter does not depend on $\lambda$:

$$\int_{-\infty}^{\infty}\left[\frac{\partial^l}{\partial\lambda^l}s(t;\ \lambda)\right]^2 dt=$$
$$=\left[\iint_{-\infty}^{\infty}\frac{\partial^l}{\partial\lambda^l}s(t_1;\ \lambda)\,\theta(t_1;\ t_2)\,\frac{\partial^l}{\partial\lambda^l}s(t_2;\ \lambda)\,dt_1dt_2\right]=$$
$$=k_l\ (l=0,\ 1,\ \ldots). \qquad (2.94)$$

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Condition, analogous (2.94), can be recorded, also, for vector parameter $\vec{\lambda}$. Subsequently we will call the parameters of signal $\vec{\lambda}$, for which is satisfied condition (2.93), substantially nonenergetic parameters, and the signal functions of form (2.93) - by stationary type signal functions. The substantially manpower parameters include the temporary/time and frequency shifts/shears, which determine range and velocity into the radar, and also other parameters, mapped into signal by means of frequency and phase (temporary/time) modulation of harmonic oscillations or periodic sequences of impulses/momenta/pulses.

Above has already been noted that in majority of practical applications/appendices received signal has random initial phase. In this case as the output effect of optimum (or sufficient) receiver instead of $q(\vec{\lambda})$ should be accepted $Q(\vec{\lambda})$. Let us assume again, that the true value of the set of the parameters $\vec{\lambda}$ in the oscillation u(t) adopted is equal to $\vec{\lambda}_н$. Then

$$Q(\vec{\lambda}) = \sqrt{q^2(\vec{\lambda}) + q_{\perp}^2(\vec{\lambda})} = \sqrt{[q_s(\vec{\lambda}_н;\ \vec{\lambda}) + q_n(\vec{\lambda})]^2 + [q_{s\perp}(\vec{\lambda}_н;\ \vec{\lambda}) + q_{n\perp}(\vec{\lambda})]^2}. \quad (2.95)$$

Functions $q_{s\perp}(\vec{\lambda}_н;\ \vec{\lambda})$ and $q_{n\perp}(\vec{\lambda})$ are determined by expressions (2.89) and (2.90), in which it is necessary to only replace $s(t;\ \vec{\lambda})$ and $s(t_2;\ \vec{\lambda})$

by $s_{\perp}(t;\vec{\lambda})$ and $s_{\perp}(t_2;\vec{\lambda})$.

Let us further introduce determinations of complex signal function

$$\dot{Q}_s(\vec{\lambda}_1;\vec{\lambda}_2)=q_s(\vec{\lambda}_1;\vec{\lambda}_2)+jq_{s\perp}(\vec{\lambda}_1;\vec{\lambda}_2)=$$
$$=\frac{1}{N_0}\int_{-\infty}^{\infty}\dot{S}(t;\vec{\lambda}_1)\dot{S}^*(t;\vec{\lambda}_2)\,dt=$$
$$=\left[\iint_{-\infty}^{\infty}\dot{S}(t_1;\vec{\lambda}_1)\theta(t_1;t_2)\dot{S}^*(t_2;\vec{\lambda}_2)\,dt_1dt_2\right], \qquad (2.96)$$

of its modular value

$$Q_s(\vec{\lambda}_1;\vec{\lambda}_2)=|\dot{Q}_s(\vec{\lambda}_1;\vec{\lambda}_2)|=\sqrt{q_s^2(\vec{\lambda}_1;\vec{\lambda}_2)+q_{s\perp}^2(\vec{\lambda}_1;\vec{\lambda}_2)} \qquad (2.97)$$

and interference component of output effect

$$Q_n(\vec{\lambda})=q_n(\vec{\lambda})\cos[\varphi(\vec{\lambda}_n;\vec{\lambda})]+q_{n\perp}(\vec{\lambda})\sin[\varphi(\vec{\lambda}_n;\vec{\lambda})], \qquad (2.98)$$

where

$$\cos[\varphi(\vec{\lambda}_n;\vec{\lambda})]=\frac{q_s(\vec{\lambda}_n;\vec{\lambda})}{Q_s(\vec{\lambda}_n;\vec{\lambda})},$$
$$\sin[\varphi(\vec{\lambda}_n;\vec{\lambda})]=\frac{q_{s\perp}(\vec{\lambda}_n;\vec{\lambda})}{Q_s(\vec{\lambda}_n;\vec{\lambda})}. \qquad (2.99)$$

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For normal work of system is required considerable excess of signal above interference: $\mu(\vec{\lambda}_n)\gg 1$. Respectively, taking into account (2.92) and utilizing the introduced definitions, instead of (2.95) we obtain

$$Q(\vec{\lambda})=\sqrt{Q_s^2(\vec{\lambda}_n;\vec{\lambda})+2Q_s(\vec{\lambda}_n;\vec{\lambda})Q_n(\vec{\lambda})+q_n^2(\vec{\lambda})+q_{n\perp}^2(\vec{\lambda})}\approx$$
$$\approx\begin{cases}Q_s(\vec{\lambda}_n;\vec{\lambda})+Q_n(\vec{\lambda}) & \text{when } \vec{\lambda}\in v(\vec{\lambda}_n),\\ \sqrt{q_n^2(\vec{\lambda})+q_{n\perp}^2(\vec{\lambda})} & \text{when } \vec{\lambda}\notin v(\vec{\lambda}_n).\end{cases} \qquad (2.100)$$

Again output effect $Q(\vec{\lambda})$ in vicinity of evaluation/estimate or of vicinity of true value of parameter $\vec{\lambda}_{u}$ is of sum of signal $Q_s(\vec{\lambda}_u; \vec{\lambda})$ and interference $Q_n(\vec{\lambda})$ of components, interference component (2.98) presenting normal random process with zero mathematical expectation and with correlation function

$$\langle Q_n(\vec{\lambda}_1) Q_n(\vec{\lambda}_2)\rangle = Q_s(\vec{\lambda}_1; \vec{\lambda}_2) \text{ with } \lambda_1, \lambda_2 \in v(\vec{\lambda}_u), \quad (2.101)$$

that coinciding in form with signal function $Q_s$. During the conclusion/output of approximate relationship/ratio (2.101) there is conducted the expansion of functions $\varphi(\vec{\lambda}_u; \vec{\lambda}_1)$ and $\varphi(\vec{\lambda}_u; \vec{\lambda}_2)$ in the power series in the vicinity of point $\vec{\lambda}_u$ with the retention/preservation/maintaining only of the linear terms of series/row. It is possible to also use approximation for the combined random number distribution $Q(\vec{\lambda}_1)$ and $Q(\vec{\lambda}_2)$ in the large ratio $\mu$ [32].

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The ratio of the power of signal component $Q_s^2(\vec{\lambda}_u; \vec{\lambda}_u)$ to the dispersion

of interference component $\langle Q_n^2(\vec{\lambda}_n)\rangle$ is equal to $2\mu(\vec{\lambda}_n)$, it coincides with (2.92).

Before drawing some practical conclusions, it is expedient still to examine form of the function of plausibility (2.42) and (2.45) in vicinity of evaluation/estimate $\vec{\lambda}_m$ in relation signal/noise, sufficient for normal work of system ($\mu$>>1). For simplification in the writing of formulas we will assume that the totality $\vec{\lambda}$ encompasses all random parameters of signal, including unessential, if they are. In the vicinity of maximum likelihood estimate $\vec{\lambda}_m$ ($\vec{\lambda}_m = \lambda_{m1}, \ldots, \lambda_{mn}$)

$$s(t;\ \vec{\lambda}) \approx s(t;\ \vec{\lambda}_m) + \sum_{i=1}^{n} (\lambda_i - \lambda_{mi}) \frac{\partial}{\partial\lambda_i} s(t;\ \vec{\lambda}_m);\ \vec{\lambda} \in v(\vec{\lambda}_m). \tag{2.102}$$

Substituting (2.102) in (2.42) and (2.45) and taking into account system of equations of plausibility

$$\frac{2}{N_0} \iint_{-\infty}^{\infty} \left[u(t) - s(t;\ \vec{\lambda}_m)\right] \frac{\partial}{\partial\lambda_i} s(t;\ \vec{\lambda}_m)\, dt =$$

$$= \left[\iint_{-\infty}^{\infty} [u(t_1) - s(t_1;\ \vec{\lambda}_m)]\, \theta(t_1;\ t_2) \frac{\partial}{\partial\lambda_i} s(t_2;\ \vec{\lambda}_m) \times \right.$$

$$\left. \times\, dt_1 dt_2 \right] = 0 \quad (i = 1, \ldots, n), \tag{2.103}$$

we obtain approximation

$$p(\vec{u}\mid\vec{\lambda}) = k \exp\left[-\frac{1}{2} \sum_{i,\ j=1}^{n} A_{ij} (\lambda_i - \lambda_{mi})(\lambda_j - \lambda_{mj})\right],$$

$$\lambda \in v(\vec{\lambda}_m), \tag{2.104}$$

where

$$A_{ij}=\frac{2}{N_0}\int_{-\infty}^{\infty}\frac{\partial}{\partial\lambda_i}s(t;\vec{\lambda}_m)\frac{\partial}{\partial\lambda_j}s(t;\vec{\lambda}_m)\,dt=$$

$$=\left[\iint_{-\infty}^{\infty}\frac{\partial}{\partial\lambda_i}s(t_1;\vec{\lambda}_m)\theta(t_1;t_2)\frac{\partial}{\partial\lambda_j}s(t_2;\vec{\lambda}_m)\,dt_1dt_2\right]. \quad (2.105)$$

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In [49] is given another useful approximation of function of plausibility:

$$p(\vec{u}\mid\vec{\lambda})=k\exp[-\mu(\vec{\lambda})+q_s(\vec{\lambda}_m;\vec{\lambda})],\ \lambda\in v(\vec{\lambda}_m), \quad (2.106)$$

which has the same level of accuracy, that also (2.104), and in region $\vec{\lambda}\in v(\vec{\lambda}_m)$ coincides with (2.104), if only there are that figuring in (2.104) derivatives. When some of the parameters $\vec{\lambda}$ are unessential, expression for the function of plausibility for the essential parameters is only obtained via averaging (2.104) and (2.106) from the unessential parameters. In particular, if totality $\vec{\lambda}$ consists of the essential parameters $\vec{\lambda}$ and one unessential - initial phase $\varphi$( $\vec{\lambda}=\vec{\lambda}$, $\varphi$), then averaging (2.106) on $\varphi$ gives

$$p(\vec{u}\mid\vec{\lambda})=k\exp[-\mu(\vec{\lambda})]\,I_0[Q_s(\vec{\lambda}_m;\vec{\lambda})],\ \vec{\lambda}\in v(\vec{\lambda}_m). \quad (2.107)$$

Consequently, in vicinity of evaluation/estimate $\vec{\lambda}_m$ function of plausibility in large relation signal/noise $\mu$, in the first place, has character of Gaussian curve (or of Gaussian surface); in the second place, it depends on realization of oscillation u(t) adopted not directly, but only inasmuch as on u(t) depends maximum likelihood estimate $\vec{\lambda}_m$.

For approximate representation in vicinity of evaluation $\vec{\lambda}_m$ of function of plausibility or other output effects of radio receiving equipment, for example functions $q(\vec{\lambda})$ and $Q(\vec{\lambda})$, in expressions, which determine these functions, should be replaced realization of oscillation u(t) adopted by signal $s(t;\ \vec{\lambda}_m)$ either $s(t;\ \vec{\lambda}_m;\ \vec{\alpha}_m)$, whose parameters $\vec{\lambda}$ or $\vec{\lambda}$, $\vec{\alpha}$, are taken as equal to parameters $\vec{\lambda}_m$ or $\vec{\lambda}_m$, $\vec{\alpha}_m$ at point of maximum plausibility. In the vicinity of evaluation/estimate the output effects $q(\vec{\lambda})$ and $Q(\vec{\lambda})$ coincide in the form with signal functions $q_s(\vec{\lambda}_m;\ \vec{\lambda})$ and $Q_s(\vec{\lambda}_m;\ \vec{\lambda})$, by somewhat displaced due to the presence of interferences in all axes $\lambda_1, \ldots \lambda_n$ relative to the true value of parameter $\vec{\lambda}_u$, relative to signal functions $q_s(\vec{\lambda}_u;\ \vec{\lambda})$ and $Q_s(\vec{\lambda}_u;\ \vec{\lambda})$, formed from actually/really received signal $s(t;\ \vec{\lambda}_u)$. From one realization to the next the bias/displacement (or error) $\vec{\lambda}_m - \vec{\lambda}_u$, is changed.

Examination given in present paragraph indicates very important role of signal functions $q_s(\vec{\lambda}_1;\ \vec{\lambda}_2)$ and $Q_s(\vec{\lambda}_1;\ \vec{\lambda}_2)$ in communication systems in accordance with completely determined signals and with signals with random initial phase. These functions determine signal component of output effect (response of system to the useful signal), correlation function of interferences at the output of system and form of quite output effects $q(\vec{\lambda})$ and $Q(\vec{\lambda})$ in the vicinity of

evaluation/estimate for the optimum systems. For studying the form of signal functions $q_s$ and $Q_s$ in the pure form, independent of their intensity, are introduced the normalized signal functions $\psi$ and $|\Psi|$, called functions of indeterminacy or correlation functions of the signal

$$\psi(\vec{\lambda}_1;\ \vec{\lambda}_2)=\frac{q_s(\vec{\lambda}_1;\ \vec{\lambda}_2)}{\sqrt{q_s(\vec{\lambda}_1;\ \vec{\lambda}_1)\,q_s(\vec{\lambda}_2;\ \vec{\lambda}_2)}} \qquad (2.108)$$

with the completely determined signals and

$$|\dot{\Psi}(\vec{\lambda}_1;\ \vec{\lambda}_2)|=\Psi(\vec{\lambda}_1;\ \vec{\lambda}_2)=\frac{|\dot{Q}_s(\vec{\lambda}_1;\ \vec{\lambda}_2)|}{\sqrt{Q_s(\vec{\lambda}_1;\ \vec{\lambda}_1)\,Q_s(\vec{\lambda}_2;\ \vec{\lambda}_2)}} \quad (2.109)$$

with the signals with the random initial phase or with the random ones by initial phase and the intensity.

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Especially wide acceptance find functions $\psi$ and $|\dot{\Psi}|$ in different investigations of systems with the substantially manpower parameters. In this case the correlation functions of signals (ambiguity function) take the form of the functions, which depend on a difference in their arguments:

$$\psi(\vec{\lambda})=\frac{\int_{-\infty}^{\infty} s(t;\ \vec{\lambda}_1)\,s(t;\ \vec{\lambda}_1+\vec{\lambda})\,dt}{\int_{-\infty}^{\infty} s^2(t;\ \vec{\lambda})\,dt} \qquad (2.110)$$

and

$$|\dot{\Psi}(\vec{\lambda})| = \Psi(\vec{\lambda}) = \frac{\left| \int\limits_{-\infty}^{\infty} \dot{s}(t;\, \vec{\lambda}_1;\, \varphi_1)\, \dot{s}^*(t;\, \vec{\lambda}_1 + \vec{\lambda}_2;\, \varphi_2)\, dt \right|}{\int\limits_{-\infty}^{\infty} |\dot{s}(t;\, \vec{\lambda};\, \varphi)|^2\, dt}. \quad (2.111)$$

Ambiguity functions $\Psi(\vec{\lambda}_1;\ \vec{\lambda}_2)$ and $\Psi(\vec{\lambda})$ are modular value of complex ambiguity functions $\dot{\Psi}(\vec{\lambda}_1;\ \vec{\lambda}_2)$ and $\dot{\Psi}(\vec{\lambda})$, which are mathematically determined by the same expressions (2.109) and (2.111), if we in them drop/omit modulus sign. Ambiguity functions are calibrated in such a way that they are equal to one with coinciding arguments $(\vec{\lambda}_1=\vec{\lambda}_2)$ in (2.108) and (2.109), and also with the equality to zero arguments $(\vec{\lambda}=0)$ in (2.110) and (2.111).

For space-time signals $s(t;\ \vec{r};\ \vec{\lambda})$ signal functions and ambiguity functions are determined analogously. Difference consists only of the fact that integrals on the time should be replaced the integrals on the time and three-dimensional coordinates $\vec{r}$.

In other words, should be replaced t by $\vec{\omega}$=t, $\vec{r}$ and $t \in (T_1,\ T_2)$ on $\vec{\omega} \in \Omega;\ \{t \in (T_1;\ T_2);\ \vec{r} \in \vec{\mathcal{L}}\}$.

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Potential possibilities of system are determined by energy relation signal/noise and form of the function of

uncertainty/indeterminacy. For the explanation of latter position, and also in order to establish/install some considerations, by which are guided during the selection the waveforms, let us examine the task first important from a fundamental point of view of distinguishing the signals, first of all of two signals.

## 2.6. Discrimination of signals.

Task of distinguishing signals simulates broad class of communication systems, and results, obtained during solution of this problem, subsequently will be extensively used. Task is formulated as follows. The ensemble of the received signals

$$s_1(t)=s(t;\ \vec{\lambda}_1;\ \alpha);\ \ldots;\ s_M(t)=s(t;\ \vec{\lambda}_M;\ \vec{\alpha}) \quad (2.112)$$

reflects finite M number of discrete/digital communications/reports $\vec{\lambda}_1, \ldots, \vec{\lambda}_M$. Values $\vec{\lambda}_1, \ldots, \vec{\lambda}_M$ are fixed/recorded and previously known, $\vec{\alpha}$ - unessential random parameters of signal. At the completely determined signals the parameters $\vec{\alpha}$ are absent (or are previously known). In this case the discrimination is called simple in contrast to the complicated discrimination, when received signals $s_1, \ldots, s_M$ contain the random unknown parameters $\vec{\alpha}$. Are assigned: the statistics of interferences $R(t_1;\ t_2)$, usually $R(t_1;\ t_2)=0.5 N_0\delta(t_1-t_2)$; a priori probabilities $p_1, \ldots, p_M$ of the impulse/transmission of all signals $s_1, \ldots, s_M$ respectively; the probability density $p(\vec{\alpha})$ of the parameter $\vec{\alpha}$. It is necessary in the

interval of observation $t \in (T_1; T_2)$ to distinguish signals, i.e., by the realization u(t) accepted to in the best way make a decision about what signal is sent, and to give evaluation/estimate to theoretically maximum qualitative indices of system.

According to presented above in situation in question must be accepted solution by maximum of a posteriori probability.

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In particular, this means that solution $s^* = s^*_i$ $(\vec{\lambda}^* = \vec{\lambda}^*_i)$ must be accepted, if the following system from M-1 inequality is satisfied:

$$Y_i \geqslant Y_j (j = 1, \dots, M;\ j \neq i), \qquad (2.113)$$

where $Y_1, \dots, Y_M$ - output effects of the optimum radio receiving equipment, which are the arbitrary monotonically increasing functions of a posteriori probability $P(\vec{\lambda}_i | \vec{u})$. For calculation $P_{omi}$ - the conditional probability of error during the transmission of the i signal - we assume that the input of system enters the oscillation, which contains i signal $(\vec{u} = \vec{s}_i + \vec{n})$, and we find first the probability of making a correct decision (probability of the satisfaction of the system of inequalities (2.113):

$$P(s^*_i | s_i) = \int_{-\infty}^{\infty} dY_i \int_{-\infty}^{\infty} \dots \int p(Y_1, \dots, Y_M | s_i) \times$$
$$\times dY_1 \dots pY_j \dots dY_M \ (j \neq i),$$

where $p(Y_1, \ldots, Y_M|s_i)$ - conditional (when $s=s_i$) combined density of the distribution of output effects $Y_1, \ldots, Y_M$. Probability of error during the transmission of the i signal

$$P_{\text{ош}\, i} = 1 - P(s^*_i | s_i) = \\ = 1 - \int_{-\infty}^{\infty} dY_i \int_{-\infty}^{\infty} \ldots \int p(Y_1, \ldots, Y_M | s_i)\, dY_1 \ldots dY_j \ldots dY_M \; (j \neq i). \quad (2.114)$$

Average/mean probability of erroneous solutions $P_{\text{ош.ср}}$, which expresses quality of system (during use of criterion of ideal observer or simple loss function), is determined by conditional probabilities of errors and by a priori probabilities:

$$P_{\text{ош ср}} = \sum_i P_i P_{\text{ош}\, i}.$$

Calculations according to formula (2.114) are sufficiently complicated and not not always feasible in final form. With the mutual statistical independence of output effects $Y_1, \ldots, Y_M$ or during the normal distribution of these values the calculations considerably are simplified.

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We will be limited by the examination of several particular, but practically most interesting cases.

Discrimination of two completely determined signals. The

signals, which are subject to discrimination,

$$s_1(t) = s(t;\ \vec{\lambda}_1) \text{ and } s_2(t) = s(t;\ \vec{\lambda}_2)$$

are completely known, have the a priori probabilities $P_1$ and $P_2$ respectively and signal function (2.89) such, that

$$q_s(\vec{\lambda}_i;\ \vec{\lambda}_j) = \begin{cases} 2\mu_1 & ^{(1)}\text{при } i=j=1, \\ 2\mu_2 & \text{при } i=j=2, \\ 2\sqrt{\mu_1\mu_2}\psi & \text{при } i\neq j;\ i,\ j=1,\ 2. \end{cases} \qquad (2.115)$$

Key: (1). with.

Coefficient of correlation $\psi$ and energy relation signal/noise $\mu$ are determined by means of (2.108) and (2.48), and utilized below functions $q(\vec{\lambda})$ and $q_n(\vec{\lambda})$ - by means of (2.47) and (2.90).

As optimum output effects let us select numbers $Y_1$ and $Y_2$,

$$Y_i = \ln P(\vec{\lambda}_i \mid \vec{u}) - \ln k = -\mu_i + q(\vec{\lambda}_i) + \ln P_i \quad (i=1,\ 2),$$

and, after supposing that the input of system signal $s_1$ ($u=s_1+n$) enters, i.e., that

$$Y_1 = \mu_1 + q_n(\vec{\lambda}_1) + \ln P_1,$$

$$Y_2 = -\mu_2 + 2\sqrt{\mu_1\mu_2}\,\psi + q_n(\vec{\lambda}_2) + \ln P_2,$$

let us find probability of satisfaction of inequality $Y_2>Y_1$ or inequality

$$q_n(\vec{\lambda}_2) - q_n(\vec{\lambda}_1) > \mu_1 + \mu_2 - 2\sqrt{\mu_1\mu_2}\,\psi + \ln\frac{P_1}{P_2}. \qquad (2.116)$$

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This probability is equal to the conditional probability of erroneous solution $P_{\text{ош1}} = P(s^*_2|s_1)$. Value $q_n(\vec{\lambda}_2) - q_n(\vec{\lambda}_1)$, which stands on the left side (2.116), is normal random variable with the zero mathematical expectation and the dispersion

$$\langle [q_n(\vec{\lambda}_2) - q_n(\vec{\lambda}_1)]^2 \rangle = 2(\mu_1 + \mu_2 - 2\sqrt{\mu_1\mu_2}\psi).$$

Consequently

$$P_{\text{ош1}} = 1 - \Phi\left[\sqrt{\frac{\mu_1 + \mu_2 - 2\sqrt{\mu_1\mu_2}\psi}{2}} + \frac{\ln\frac{P_1}{P_2}}{\sqrt{2(\mu_1 + \mu_2 - 2\sqrt{\mu_1\mu_2}\psi)}}\right]. \quad (2.117)$$

Here and subsequently $\Phi(x)$ - probability integral

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{t^2}{2}} dt.$$

It is analogous,

$$P_{\text{ош2}} = 1 - \Phi\left[\sqrt{\frac{\mu_1 + \mu_2 - 2\sqrt{\mu_1\mu_2}\psi}{2}} + \frac{\ln\frac{P_2}{P_1}}{\sqrt{2(\mu_1 + \mu_2 - 2\sqrt{\mu_1\mu_2}\psi)}}\right]$$

and

$$P_{\text{ош ср}} = P_1 P_{\text{ош1}} + P_2 P_{\text{ош2}}.$$

With equality of a priori probabilities

$$P_1 = P_2 = 0.5 \quad (2.118)$$

conditional probabilities of errors $P_{\text{ош}1}$ and $P_{\text{ош}2}$ are equal to each other,

$$P_{\text{ош}1}=P_{\text{ош}2}=P_{\text{ош}},$$

and they coincide with average/mean probability of erroneous solutions $P_{\text{ош ср}}$:

$$P_{\text{ош ср}}=P_{\text{ош}}=1-\Phi\left[\sqrt{\frac{\mu_1+\mu_2-2\sqrt{\mu_1\mu_2}\psi}{2}}\right] \quad (2.119)$$

independent of relationship/ratio between energies of signals.

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Obtained result (2.119) can be also represented in the form

$$P_{\text{ош ср}}=P_{\text{ош}}=1-\Phi\left(\sqrt{\frac{\mu_{\text{экв}}}{2}}\right), \quad (2.120)$$

where $\mu_{\text{экв}}$ - energy relation signal/noise for equivalent signal

$$s_{\text{экв}}(t)=s_1(t)-s_2(t), \quad (2.121)$$

and to interpret so that average/mean probability of erroneous solutions on this interference level depends only on energy of equivalent signal. From (2.120) and (2.121) it is possible to draw a useful conclusion. If it is necessary to distinguish two signals $s_2(t)$ and $s_2(t)$, that coincide with each other in some time intervals, then for the discrimination only the noncoincident part of the signals is utilized, whereas energy of the coinciding part is lost uselessly.

With fixed/recorded medium energy of signals

$$\mu=P_1\mu_1+P_2\mu_2=0.5(\mu_1+\mu_2)=\text{const}$$

smallest possible value of mean error

$$P_{\text{out cp}} = P_{\text{out}} = 1 - \Phi(\sqrt{2\mu}) \qquad (2.122)$$

is obtained with $\mu_1=\mu_2=\mu$ and $\psi=-1$, i.e., with opposite signals $s_2(t)=-s_1(t)$, characteristic for binary systems with phase manipulation.

With orthogonal signals ($\psi=0$), characteristic for binary systems with frequency ($\mu_1=\mu_2=\mu$) and amplitude ($\mu_1=2\mu$; $\mu_2=0$) manipulation, average/mean probability of error

$$P_{\text{out cp}} = P_{\text{out}} = 1 - \Phi\left(\sqrt{\frac{\mu_1+\mu_2}{2}}\right) = 1 - \Phi(\sqrt{\mu}) \qquad (2.123)$$

depends on this interference level only on medium energy of signals $\mu$, but not on how this energy is distributed between signals $s_1$ and $s_2$. From the given relationships/ratios it follows that the use/application of opposite signals instead of the orthogonal ones makes it possible two times to decrease the energy of signals (or relation $\mu$) with the retention/preservation/maintaining of the constant/invariable probability of erroneous solutions.

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Obtained formulas (2.122) and (2.123) can be also used for calculation according to given one for this system of maximum

permissible average/mean probability of erroneous solutions $P_{\text{ош доп}}$, by assigned reliability of discrimination of signals, minimally distinguishable relation signal/noise $\mu_{\text{мин}}$, called also threshold. Thus, for instance, from (2.123) we find the minimally distinguishable relation signal/noise for the systems with the orthogonal signals:

$$\mu_{\text{мин}} = [\operatorname{arc} \Phi(1 - P_{\text{ош доп}})]^2. \quad (2.124)$$

Here and subsequently symbol arc designates inverse function; so that if $y=\Phi(x)$, then function arc $\Phi$ expresses dependence of x on y.

Possibility of substantiated calculation of figuring in formulas of range of radio communication minimally distinguishable (threshold) relations signal/noise or threshold signals is one of fundamental results of statistical theory of connection/communication.

For obtaining comparatively reliable discrimination of orthogonal signals $s_1$ and $s_2$ from $P_{\text{ош}}$ order $10^{-3}$ and less according to (2.124) energy relation signal/noise must be $\mu \geq 9.6$. The latter confirms one of the initial assumptions of the theory of the estimations of the parameters of signal about the considerable excess of the signal above the interference under working conditions of the functioning of systems, since the evaluation/estimate of the discrete/digital parameters is identical to discrimination, and the estimate of the continuous parameters can be treated as the

discrimination of the correlated (differing little on $\vec{\lambda}$) signals. In this case one should emphasize that the discussion deals with the energy excess of the signal above interference ($\mu$>>1), whereas relation signal/noise according to power input of system can be any and, in particular, smaller than one.

Discrimination of two signals with random initial phases. During the discrimination of signals $s_1$ and $s_2$ with the random initial phases (incoherent signals)

$$s_1(t) = s(t;\ \vec{\lambda}_1;\ \varphi) = s(t;\ \vec{\lambda}_1)\cos\varphi + s_\perp(t;\ \vec{\lambda}_1)\sin\varphi,$$

$$s_2(t) = s(t;\ \vec{\lambda}_2;\ \varphi) = s(t;\ \vec{\lambda}_2)\cos\varphi + s_\perp(t;\ \vec{\lambda}_2)\sin\varphi$$

the measure for the correlation between the signals is the coefficient $\Psi$, determined by means of (2.109) and (2.111).

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Conclusion/output of approximation for average/mean probability of erroneous solutions $P_{\text{ош ср}}$ in this case virtually repeats recently made conclusion/output and leads to analogous expressions, in which only one should replace $\psi$ by $\Psi$. For shortening of linings/calculations let us examine the fundamental case, when signals $s_1$ and $s_2$ have the identical energy

$$Q_s(\vec{\lambda}_i;\ \vec{\lambda}_j) = \begin{cases} 2\mu & \text{при } i = j, \\ 2\mu\Psi & \text{при } i \neq j\ (i, j = 1, 2) \end{cases} \qquad (2.125)$$

Key: (1). with.

and the identical a priori probabilities $P_1=P_2=0.5$.

Then as optimum output effect it is possible to accept values $Q(\vec{\lambda}_1)$ and $Q(\vec{\lambda}_2)$.. The conditional probability of erroneous solution during the transmission of first signal $s_1—P_{\text{ош1}}$ in this case will be equal to the probability of the satisfaction of inequality $Q(\vec{\lambda}_2)>Q(\vec{\lambda}_1)$, which, taking into account (2.100), (2.101), and (2.125), is converted in

$$\hat{Q}_n<2\mu(1-\Psi),$$

where $\hat{Q}_n$ - normal random variable with the zero mathematical expectation and the dispersion

$$\langle\hat{Q}_n^2\rangle=\langle[Q_n(\vec{\lambda}_2)-Q_n(\vec{\lambda}_1)]\rangle=4\mu(1-\Psi).$$

Consequently,

$$P_{\text{ош ср}}=P_{\text{ош}}=P_{\text{ош1}}=1-\Phi[\sqrt{\mu(1-\Psi)}]. \quad (2.126)$$

The comparison of approximation formula (2.126) with results of precise calculations [65] makes it possible to conclude that (2.126) it provides accuracy, sufficient for technical ones application/appendix, with $\mu\Psi\geq2$. For incoherent orthogonal signals (2.126) a somewhat decreased value of the probability of errors gives $P_{\text{ош}}$. However, in the case of the incoherent uncorrelated signals it is easy to conduct calculation $P_{\text{ош}}$ in the final form according to fundamental formula (2.114), which gives

$$P_{\text{ош}}=\frac{1}{2}\exp\left(-\frac{\mu}{2}\right). \quad (2.127)$$

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Derivation of latter/last formula is omitted, since it is special case of given below expression for probability of errors in system with M by orthogonal signals. On the basis of (2.126) it is possible to draw the conclusion that for distinguishing two correlated signals with the random initial phases only the uncorrelated (1-Ψ) part of the energy of signals is utilized, whereas the correlated part (Ψ - part of total energy of signal) it is lost uselessly.

Discrimination M of orthogonal signals. We will assume that all signals, entering the ensemble of transmitted signals (2.112), are equally probable to:

$$P_1 = \ldots P_M = \frac{1}{M}, \qquad (2.128)$$

have identical energies and is mutually orthogonal (they are not correlated), i.e.,

$$\frac{2}{N_0}\int_{-\infty}^{\infty} s_i(t)\, s_j(t)\, dt =$$
$$= \left[\iint_{-\infty}^{\infty} s_i(t_1)\, \theta(t_1; t_2)\, s_j(t_2)\, dt_1 dt_2\right] = \begin{cases} 2\mu & \text{при } i = j, \\ 0 & \text{при } i \neq j. \end{cases} \qquad (2.129)$$

Key: (1). with.

Optimum output effects $Y_1, \ldots, Y_M$ are arbitrary monotonically increasing functions $\varphi$ of likelihood ratio

$$Y_i = \varphi \left\{ \int_{\Lambda} p(\vec{\alpha}) \exp \left[ -\mu + \int_{-T}^{\infty} u(t)\, s(t;\ \vec{\lambda}_i;\ \vec{\alpha})\, dt \right] d\vec{\alpha}. \right. \quad (2.130)$$

In view of mutual orthogonality of signals the normal random variables $\int u(t) s(t;\ \vec{\lambda}_i;\ \vec{\alpha}) dt$, entering in (2.130), are statistically independent with different values of index i. Therefore also are statistically independent output effects $Y_i$, $Y_j$ with i≠j. Furthermore, during the transmission of signal $s_i$ $(u=s_i+n)$ all output effects $Y_j$ (j≠i) have one and the same law of distribution $p_N(Y_j)$ - the distribution law for j output effect $Y_j$ in the absence on the input of j signal $s_j$. The law of distribution of output effect $Y_i$ when, at the input, signal is present, $s_i$ let us designate $p_{SN}(Y_i)$

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Functions $p_N(Y)$ and $p_{SN}(Y)$ subsequently are called the laws of distribution of output effect in the absence and presence of signal respectively. Taking into account the aforesaid, the conditional combined density of distribution $p(Y_1, \ldots, Y_M | s_i)$, entering in (2.114), can be represented in the form

$$p(Y_1, \ldots, Y_M | s_i) = p_{SN}(Y_i)\, p_N(Y_1) \ldots p_N(Y_j) \ldots p_N(Y_M) (j \neq i).$$

and the average/mean probability of errors during discrimination M of mutually orthogonal signals is equal to

$$P_{\text{ош ср}} = P_{\text{ош}} = 1 - \int_{-\infty}^{\infty} p_{SN}(Y)\, dY \left[ \int_{-\infty}^{Y} p_N(x)\, dx \right]^{M-1}. \tag{2.131}$$

With reception of completely determined signals as optimum output effect it is possible to accept

$$Y_i = \frac{2}{N_0} \int_{-\infty}^{\infty} u(t)\, s_i(t)\, dt =$$
$$= \left[ \iint_{-\infty}^{\infty} u(t_1)\, \theta(t_1;\, t_2)\, s_i(t_2)\, dt_1 dt_2 \right]; \quad (i = 1, \ldots, M). \tag{2.132}$$

Output effects (2.132) are normal random variables with following distribution laws:

$$p_N(Y) = \frac{1}{\sqrt{2\pi 2\mu}} \exp\left(-\frac{Y^2}{2 \cdot 2\mu}\right), \tag{2.133}$$
$$p_{SN}(Y) = \frac{1}{\sqrt{2\pi 2\mu}} \exp\left[-\frac{(Y - 2\mu)^2}{2 \cdot 2\mu}\right]. \tag{2.134}$$

Substituting (2.133) and (2.134) in (2.131), we obtain

$$P_{\text{ош ср}} = P_{\text{ош}} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp\left[\frac{(Y - \sqrt{2\mu})^2}{2}\right] [\Phi(Y)]^{M-1} dY. \tag{2.135}$$

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Integral (2.135) cannot be expressed in elementary functions. In §4.4 will be given the approximate method of its computation.

With reception of signals with unknown random phase as optimum output effect it is possible to accept values

$$Y_i = \left| \frac{1}{N_0} \int_{-\infty}^{\infty} \dot{u}(t)\, \dot{s}^*_i(t)\, dt \right| =$$
$$= \left[ \frac{1}{2} \left| \iint_{-\infty}^{\infty} \dot{u}(t_1)\, \theta(t_1;\, t_2)\, \dot{s}^*_i(t_2)\, dt_1 dt_2 \right| \right], \qquad (2.136)$$

having in the case absences and presence of signal following distributions:

$$p_N(Y) = \frac{Y}{2\mu} \exp\left(-\frac{Y^2}{2 \cdot 2\mu}\right), \qquad (2.137)$$
$$p_{SN}(Y) = \frac{Y}{2\mu} \exp\left(-\frac{Y^2 + 4\mu^2}{2 \cdot 2\mu}\right) I_0(Y). \qquad (2.138)$$

Substitution (2.137) and (2.138) in (2.131) gives [54]

$$P_{\text{ош ср}} = P_{\text{ош}} = \sum_{i=1}^{M-1} (-1)^{i+1} C^i_{M-1} \frac{1}{i+1} \exp\left(-\frac{i}{i+1}\mu\right). \qquad (2.139)$$

During discrimination of two orthogonal incoherent signals latter/last formula is converted in (2.127).

2.7. Dependence of qualitative indices of system on the ambiguity function of signal.

Let us assume, evaluation/estimate of substantially manpower parameter $\vec{\lambda}$, which belongs to assigned a priori interval $\Lambda$, is conducted. Assumption about the manpower character of the parameter is done for simplification in the reasonings and is not necessary.

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In this case optimum output effect will be the additive mixture of interference and signal components of form $2\mu\psi(\vec{\lambda}_{u}-\vec{\lambda})$ or $2\mu\Psi(\vec{\lambda}_{u}-\vec{\lambda})$, "amplitude" of which is determined by energy relation signal/noise $\mu$, and form - by form of correlation function $\psi$ or $\Psi$ depending on whether the previously initial phase of received signals with each this value $\vec{\lambda}$ is known or not known. Through $\vec{\lambda}_{u}$ the markedly true value of the measured parameter $\vec{\lambda}$. In the majority to the communication system the initial phase of received signals is the random unessential parameter. Therefore with tne presentation we will be oriented toward the function $\Psi$, bearing in mind that everything said can be referred to the communication systems with the completely determined signals and respectively to the ambiguity function $\psi$.

Measurement of parameter $\vec{\lambda}$ is reduced to determination against

the background of output interferences of point, in which signal function or ambiguity function reaches their greatest value. Therefore the accuracy of the evaluation/estimate of the parameter, other conditions being equal, will be higher, the sharper is expressed the maximum of ambiguity function, i.e., is sharper/more acute and the narrower the overshoot (peak) of function $\Psi(\vec{\lambda})$ in vicinity of zero.

There is greatest interest in connection/communication of form of ambiguity function with system resolution. For the permission/resolution of signals with the parameters $\vec{\lambda}_1$ and $\vec{\lambda}_2$, which differ little from each other, signal components of output effects or function $\Psi(\vec{\lambda}-\vec{\lambda}_1)$ and $\Psi(\vec{\lambda}-\vec{\lambda}_2)$ must not highly overlap. Consequently, resolution is also obtained higher, the narrower the overshoot of function $\Psi(\vec{\lambda})$ in vicinity of zero. However, vital importance takes the form of uncertainty/indeterminacy not only in vicinity of zero. Ambiguity function $\Psi(\vec{\lambda})$ in the interval of the possible values of the evaluated parameters must have one sharply pronounced overshoot. In the presence of the supplementary intense overshoots, which approach in their amplitude the main bang pipe, any of the maximums of ambiguity function (or output effect) can be accepted as the evaluation/estimate, as a result of which the ambiguity of the evaluation/estimate of the parameter appears.

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Furthermore, the supplementary overshoots of function of the uncertainty/indeterminacy of the strong interfering signals can be applied on the main bang pipe of weaker useful signal and completely disguise it. This means that the supplementary overshoots of ambiguity function worsen/impair resolution by large differences in the value of the essential parameter $\vec{\lambda}$.

Given reasonings indicate that depending on designation/purpose of communication system and on requirements, presented to its qualitative indices, must be produced corresponding requirements for form of ambiguity function, in particular to width and sharpness of main overshoots to level of supplementary overshoots. Analogous conclusions, but with the clearer explanation of the essence of the limitations, imposed by the form of the function of uncertainty/indeterminacy to the resolution of system, it is possible to draw, on the basis of results, obtained in the examination of the task of distinguishing two signals.

For correlated signals with random initial phase or completely determined signals with $\mu_1=\mu_2=\mu$ threshold relation signal/noise corresponding to assigned average/mean probability of erroneous solutions $p_{\text{ош ср.}}$ according to (2.126) and (2.119) is equal

$$\mu_{\text{мин}} = \frac{[\text{arc}\,\Phi\,(1 - P_{\text{ош зад}})]^2}{1 - \psi}.$$

$$\mu_{\text{мин}} = \frac{[\text{arc}\,\Phi\,(1 - P_{\text{ош зад}})]^2}{1 - \psi}. \qquad (2.140)$$

From (2.140) it follows that in principle it is possible to distinguish or to solve any two signals (in sense, explained in S1.4), i.e., that theoretical boundary of permission/resolution there does not exist. However, the presence of correlation between the signals increases energy of threshold signals $\frac{1}{1-\varrho}$ once.

Correlation between signals can be caused by the fact that difference between parameters of these signals $\vec{\lambda}_1-\vec{\lambda}_2$ is small and falls into region of main bang pipe of ambiguity function, and also fact that difference $\vec{\lambda}_1-\vec{\lambda}_2$, although it is comparatively great, falls into region of supplementary overshoot of ambiguity function.

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In proportion to the approximation/approach of the coefficient of correlation $\Psi$ to one the energy of signals minimally necessary for the reliable permission/resolution sharply grows/rises and signals become virtually they are not distinguished from each other (or they are irresolvable). In this sense it is possible to assert that for

reliable permission/resolution of two and more than signals the coefficient of correlation of these signals should be substantially less than one or, in connection with signals with the random initial phases, signals must be virtually not correlated (they are orthogonal).

Finally, form of the function of uncertainty/indeterminacy to a considerable extent affects diagram of optimum working/treatment, which must reproduce module value of correlation integral $Q(\vec{\lambda})$ or any other informational equivalent of function of plausibility. Let us assume first, that the measured parameter λ is scalar and belongs to the a priori interval of possible values $(\lambda_{мин}; \lambda_{макс})$. Optimum system in the general case is constructed according to the multichannel diagram (Fig. 2.1) and reproduces in the a priori interval instead of continuous function $Q(\lambda)$ the totality of its discrete/digital values $Q(\lambda_1), \ldots, Q(\lambda_m)$. Obviously, a number of channels of working/treatment m must be selected by such so that with any true value of the measured parameter at least in one channel the sharp overshoot of signal component of output effect would occur. In other words, "distance" between adjacent channels $\lambda_i - \lambda_{i-1}$ must be not less than the width of the fundamental peak of ambiguity function $\Psi(\lambda)$, which is called the width of the region of high correlation. They count off the width of the region of high correlation $\lambda_{кор}$ at the caused level (usually 0.5-0.7) or determine by the relationship/ratio

$$\lambda_{кор} = \int_{-\infty}^{\infty} |\Psi(\lambda)|^2 d\lambda. \qquad (2.141)$$

Thus, is minimally necessary number of channels in diagram of optimum working/treatment equally

$$m = \frac{\lambda_{макс} - \lambda_{мин}}{\lambda_{кор}}. \qquad (2.142)$$

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In many technical applications/appendices, connected with approximate computations, ambiguity function Ψ(λ) it is convenient to approximate by function of form

$$\Psi(\lambda) = \begin{cases} 1 \text{ (1) в области высокой корреляции,} \\ 0 \text{ (2) вне области высокой корреляции.} \end{cases} \qquad (2.143)$$

Key: (1). in the region of high correlation. (2). out of region of high correlation.

Signals, which have correlation function (2.143) and differing from each other in parameter λ are less than to width of region of high correlation, they are not distinguished. Respectively a number of solvable signals in the a priori interval or a number of solvable elements/cells of the space of communications is equal to $m\left(\frac{\lambda_{макс} - \lambda_{мин}}{\lambda_{кор}}\right)$. Since the signals, which cannot be distinguished of each other, it does not have sense to transmit, then, being based on

representation (2.143), real system with a continuous change in the parameter $\lambda$ approximately can be replaced with discrete/digital system with the mutually orthogonal signals, whose number is equal to a number of solvable elements/cells of the space of communications/reports.

In perfect analogy in vector parameter $\vec{\lambda}=\lambda_1, \ldots, \lambda_n$, that belongs to a priori interval $\Lambda:\{\lambda_1\in(\lambda_{1\text{мин}};\ \lambda_{1\text{макс}}), \ldots, \lambda_n\in(\lambda_{n\text{ мин}};\ \lambda_{n\text{ макс}})\}$. it is possible to determine width of region of high correlation $\lambda_{i\text{кор}}$ for each parameter $\lambda_i$. For this it is necessary to assign the reference level of the region of high correlation or to use the relationship/ratio, analogous (2.141),

$$\lambda_{i\text{ кор}}=\int_{-\infty}^{\infty}|\Psi(0;\ldots;\lambda_i;\ldots;0)|^2\,d\lambda_i. \qquad (2.144)$$

Number $m_i$ of permitted signals from arbitrary parameter $\lambda_i$ is equal to $\frac{\lambda_{i\text{ макс}}-\lambda_{i\text{ мин}}}{\lambda_{i\text{ кор}}}$. Total $m$ number of the solvable elements/cells of the space of communications/reports or number of mutually orthogonal signals, for which it is possible to approximately replace the continuous ensemble of signals, is equal to

$$m=m_1\ldots m_n=\frac{\lambda_{1\text{макс}}-\lambda_{1\text{мин}}}{\lambda_{1\text{кор}}}\cdots\frac{\lambda_{n\text{ макс}}-\lambda_{n\text{ мин}}}{\lambda_{n\text{ кор}}}. \qquad (2.145)$$

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Minimally necessary number of channels in system of optimum

working/treatment is also determined by formula (2.145).

Consequently, fundamental qualitative indices of system - resolution or number of solvable elements/cells of space of communications/reports on each of measured parameters, accuracy and uniqueness of evaluation of parameters of signal - are determined by form of the function of uncertainty/indeterminacy $\Psi$ (or $\psi$) and do not depend directly on waveform, i.e., from form of the function $s(t, \vec{\lambda})$. Signal must be selected in such a way that its ambiguity function would provide qualitative indices required from the system. The task of the selection of waveform actually is reduced to the solution of the integral equation

$$\frac{1}{2E}\left|\int_{-\infty}^{\infty}\dot{s}(t;\vec{\lambda}_1)\,\dot{s}^*(t;\vec{\lambda}_2)\,dt\right| =$$
$$=\left[\frac{1}{2\mu}\left|\iint_{-\infty}^{\infty}\dot{s}(t_1;\vec{\lambda}_1)\,\theta(t_1;t_2)\,\dot{s}^*(t_2;\vec{\lambda}_2)\,dt_1dt_2\right|\right]=$$
$$=|\Psi(\vec{\lambda}_1-\vec{\lambda}_2)|. \qquad (2.146)$$

right side of which is the assigned function. The solution of equation (2.146), of course, exists not for any functions $|\Psi(\vec{\lambda})|$. Physical consideration and information about the solution of equation (2.146) for a series/row of special cases help to overcome the difficulties, connected with the virtually reasonable assignment to ambiguity function and the determination of the corresponding waveform.

Given in present paragraph positions, characterizing dependence of potential qualitative indices of system on form of ambiguity function, are well realized and extensively they are used in radar. Considerably less they are utilized with analysis and synthesis of other communication systems despite the fact that the ambiguity functions retain their significance for any systems of the evaluation/estimate of the parameters both continuous, and discrete/digital.

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2.8. Ambiguity function in the time and the frequency.

In statistical theory of radar extensively is used ambiguity function $\Psi(\tau; \Phi)$ on temporary displacement $\tau$ and (to Doppler) effect $\Phi$ of carrier frequency $f_0$ of signal. If the emitted signal

$$\dot{s}(t) = \dot{S}(t)e^{j2\pi f_0 t} \qquad (2.147)$$

has comparatively small duration, then the echo signal, taken from this fixed/recorded, previously known direction, it is possible to represent in the form of the function

$$\dot{s}(t; \tau; \Phi; \varepsilon; \varphi) = \varepsilon\dot{S}(t-\tau) \times \\ \times \exp\{j[2\pi f_0(t-\tau) - 2\pi\Phi t + \varphi]\} \qquad (2.148)$$

time and set of the random, but constant parameters: essential $\tau$ and $\Phi$, that determine range R and radial velocity v of target, and unessential $\varphi$ and $\epsilon$ - initial phase and the parameter of intensity. From expression (2.148) for the received signal it directly follows that the measured parameters $\tau$ and $\Phi$ are substantially nonenergetic and that the signal has random initial phase $\varphi$. Respectively the ambiguity function of the signal, taken against the background of the interferences, approximated by white noises, is expressed by means of (2.111):

$$\Psi(\tau;\Phi) = |\dot{\Psi}(\tau;\Phi)| = \\ = C\left|\int_{-\infty}^{\infty} \dot{S}(t;\tau_1;\Phi_1;\varepsilon_1;\varphi_1)\,\dot{S}^*(t;\tau_2;\Phi_2;\varepsilon_2\,\varphi_2)\,dt\right|. \quad (2.149)$$

where

$$\tau = \tau_2 - \tau_1;\ \Phi = \Phi_2 - \Phi_1. \quad (2.150)$$

Here and in the future C - the factor, selected from the conditions for the standardization

$$\Psi(0;\ 0) = 1. \quad (2.151)$$

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Taking into account (2.148), expression for $\Psi(\tau, \Phi)$, can be represented in the form

$$|\Psi(\tau;\Phi)| = C\int_{-\infty}^{\infty} S(t)\,S^*(t-\tau)\,e^{j\Phi t}\,dt \quad (2.152)$$

or, introducing into examination spectrum of complex signal amplitude envelope

$$G(f)=\int_{-\infty}^{\infty}\dot{S}(t)e^{-j2\pi ft}dt, \qquad (2.153)$$

in the form

$$|\dot{\Psi}(\tau;\Phi)|=C\left|\int_{-\infty}^{\infty}G^{*}(f)G(f-\Phi)e^{j2\pi f\tau}df\right|. \qquad (2.154)$$

Relationships/ratios (2.152) and (2.154) determine also complex ambiguity functions $\dot{\Psi}(\tau, \Phi)$. For this in the relationships indicated it is necessary to drop/omit modulus signs. Function of uncertainties/indeterminacies $\Psi(\tau, \Phi)$, and also the function $\Psi(\tau)$ $=\psi(\tau; 0)$, called usually the correlation function of signal, for the fundamental radar signals are studied very in detail. The fundamental information about these functions, necessary for the following presentation, here is given without the detailed conclusions.

Overall limitation, superimposed on ambiguity function of radar signals, is known by the name of "uncertainty principle": complete combined uncertainty/indeterminacy of parameters of signal $\tau$ and $\Phi$, measured by space of body of uncertainty/indeterminacy, i.e., by space, included under surface $|\dot{\Psi}(\tau; \Phi)|^2$, does not depend on waveform and it is always equal to one

$$\iint_{-\infty}^{\infty}|\dot{\Psi}(\tau;\Phi)|^{2}d\tau d\Phi=1. \qquad (2.155)$$

what is directly checked by substitution in (2.155) expressions (2.152).

According to (2.152) and (2.154) complex ambiguity function $\dot{\Psi}(\tau, \Phi)$ with an accuracy to constant coefficient is Fourier transform from $\dot{S}(t)\dot{S}^*(t-\tau)$, and also from $\dot{G}^*(f)\dot{G}(f-\Phi)$.

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Therefore, if they have two signals $s_1(t)$ and $s_2(t)$ with complex envelopes $\dot{S}_1(t)$, $\dot{S}_2(t)$ and by complex ambiguity functions $\dot{\Psi}_1(\tau; \Phi)$, $\dot{\Psi}_2(\tau; \Phi)$ respectively, then ambiguity functions $\dot{\Psi}_{12}(\tau, \Phi)$ the composite/compound signals $s_{12}(t)$ will be equal to [4]: for signal $\dot{S}_{12}(t)=\dot{S}_1(t)\dot{S}_2(t)$

$$\dot{\Psi}_{12}(\tau; \Phi)=C\int_{-\infty}^{\infty}\dot{\Psi}_1(\tau; x)\,\dot{\Psi}_2(\tau; \Phi-x)\,dx \qquad (2.156)$$

and for the signal

$$\dot{S}_{12}(t)=\int_{-\infty}^{\infty}\dot{S}_1(x)\,\dot{S}_2(t-x)\,dx$$

$$\dot{\Psi}_{12}(\tau; \Phi)=C\int_{-\infty}^{\infty}\dot{\Psi}_1(x; \Phi)\,\dot{\Psi}_2(\tau-x; \Phi)\,dx. \qquad (2.157)$$

Furthermore, during some conversions it is useful to have in mind that ambiguity function $\dot{\Psi}_3(\tau; \Phi)$ of idealized periodic signal

$$S_3(t)=\sum_{i=-\infty}^{\infty}\delta(t-iT_0) \qquad (2.158)$$

takes form

$$\Psi_{\delta}(\tau;\Phi)=C\sum_{i=-\infty}^{\infty}\delta(\tau-iT_0)\sum_{l=-\infty}^{\infty}\delta\left(\Phi-\frac{l}{T_0}\right). \qquad (2.159)$$

Let us further give expressions for ambiguity functions of some real signals and let us make series/row of practical conclusions/outputs. First let us examine signal in the form of single radio pulse, in the general case linearly of frequency-modulated (LFM). The shape of the envelope of impulse/momentum/pulse has secondary value and is received as bell (Gaussian)

$$\dot{S}(t)=S_0\exp\left(-\frac{\pi t^2}{2T^2}\pm j2\pi\frac{F_m}{T}t^2\right). \qquad (2.160)$$

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In this case the spectrum of complex envelope is expressed by the formula

$$G(f)=\frac{\sqrt{2}TS_0}{\sqrt{1\pm j2d}}\exp\left(-\frac{2\pi T^2f^2}{1\pm j2d}\right). \qquad (2.161)$$

where T - effective duration of signal,

$$T=\frac{1}{S_0^2}\int_{-\infty}^{\infty}|\dot{S}(t)|^2\,dt; \qquad (2.162)$$

$\frac{2F_m}{T}$ - rate of change in the frequency,

$$d = 2F_m T. \qquad (2.163)$$

Effective width of spectrum of signal is equal to

$$F = \frac{1}{|G(0)|^2} \int_{-\infty}^{\infty} |G(f)|^2 df = \frac{\sqrt{1+4d^2}}{2T}. \qquad (2.164)$$

If frequency modulation is absent, then d=0 and

$$FT = 0,5. \qquad (2.165)$$

The utilized here and subsequently concepts of effective duration (2.162) and effective width of spectrum (2.164) approximately/exemplarily coincide for the bell curves with the appropriate parameters, counted off at the level - 3 dB.

With broadband frequency modulation

$$d \gg 1. \qquad (2.166)$$

According to (2.164) and (2.166)

$$d \approx FT \gg 1. \qquad (2.167)$$

Signals with product of width of spectrum to duration FT, substantially exceeding one, i.e., those subordinating to condition (2.167), call serrated signals. Signals with product FT of the order

of one, in particular pulse signals with the unmodulated filling, are called simple.

Ambiguity function of signal (2.160) is equal to

$$\Psi(\tau;\Phi)=\exp\left[-\pi\left(\frac{1+4d^2}{4}\frac{\tau^2}{T^2}+T^2\Phi^2+2d\Phi\tau\right)\right]. \quad (2.168)$$

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For approximate graphic representation on plane $\tau$, $\Phi$ ambiguity functions $\Psi(\tau;\Phi)$ it is convenient to use ducts/contours from different by denseness shading. During the qualitative discussion of different laws sufficiently good approximation/approach can be obtained by the representation of function $\Psi(\tau;\Phi)$ by means of altogether only two regions: the region of high correlation ($\Psi\geq 0.5-0.7$) and uncorrelated region. This representation corresponds to approximation (2.143) and for function (2.168) is given in Fig. 2.8. The region of high correlation, the determining extent in the appropriate direction of function or body of uncertainty/indeterminacy, is shaded.

Law, which follows from formulas (2.168) and Fig. 2.8, is general/common/total and consists of following. The width of ambiguity function along the temporary/time axis $\tau$ (width of

correlation function $\Psi(\tau; 0)$) is approximately/exemplarily equal to $1/F$, while the width of ambiguity function along the axis of frequencies f (width of function $\Psi(0, \Phi)$) is approximately/exemplarily equal to $1/T$, where F and T - width of the spectrum and duration of signal. In this case the received signal is proposed by coherent, i.e., by such that, during the assigned communication/report (assigned parameters $\tau$ and $\Phi$) and to the assigned initial phase of the oscillations/vibrations of signal at certain arbitrary moment of time, is uniquely determined the phase of oscillations/vibrations in entire interval of the existence of signal.

From (2.168) it also follows that complication of phase structure of signal, which is evinced by use/application of linear frequency modulation, leads to decrease of extent of region of high correlation, i.e., to shortening (compression) of signal component of optimum output effect 2d once.

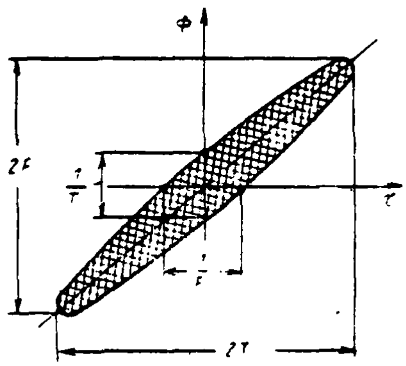


Fig. 2.8. Conditional image of ambiguity function Ψ(τ, Φ) impulse/momentum/pulse linear ChM.

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As a result so many times increases the resolution of system in the range (on the time). Value 2d (or d) is frequently called contraction coefficient. The signals, linearly frequency-modulated, are in sufficient detail examined in [45, 58, etc.].

Among serrated signals together with impulses/momenta/pulses, linearly modulated in frequency, extensively are used phase-keyed (FM) signals, which are constructed according to following principle. Interval of time T, which is the duration of signal, is divided/marked off into M of time intervals each by duration $\Delta=T/M$. The amplitude of the oscillations of signal is the constant value, identical in all time intervals. The initial phase of oscillations in

each time interval (or phase jump upon transfer from one interval to another) can accept one of two values: 0 or $\pi$. In other words, initial phase $\varphi_i$ in the arbitrary i interval (or position) is equal to

$$\varphi_i = a_i\pi;\ a_i = 0 \text{ or } 1. \qquad (2.169)$$

Respectively complex signal amplitude envelope takes form

$$\dot{S}(t) = \sum_{i=1}^{M} S_\Delta[t-(i-1)\Delta]\, e^{ja_i\pi}, \qquad (2.170)$$

where

$$S_\Delta(t) = \begin{cases} 1 \text{ (1) при } t \in (0, \Delta), \\ 0 \text{ (1) при } t \bar{\in} (0, \Delta). \end{cases} \qquad (2.171)$$

Key: (1). with.

Change in phase is conducted on previously comprised code, by means of which is established binary sequence of numbers $a_1, \ldots, a_M$, which determines signal (2.170). Effective width of the spectrum of signal $F \approx 1/\Delta$. The phase-keyed signals belong to a broader class of pseudorandom or noise-like signals [12, 45, 63]. The ambiguity function of these signals $\Psi(\tau; \Phi)$ with the sufficiently large product of duration to the width of spectrum d or with a sufficiently large number of intervals M takes the almost ideal form, represented in Fig. 2.9.

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Main overshoot of function $\Psi(\tau;\ \Phi)$, or region of high correlation, is arranged/located on plane $\tau$; $\Phi$ in vicinity of zero and has extent, approximately/exemplarily equal to $\Delta$ (or $1/F$) along axis $\tau$ and $1/T$ - along axis $\Phi$. Basic part (approximately/exemplarily $1-1/d$) of the single space of the body of uncertainty/indeterminacy is distributed in the form of comparatively small supplementary overshoots on the broad band $|\tau| \leq T$; $|\Phi| \leq F$.

If maximum frequency shift $\Phi_{\text{макс}}$ with respect to a priori predicted frequency of received signal is less $1/T$,

$$\Phi_{\text{макс}} < \frac{1}{T}, \tag{2.172}$$

then it is sufficient to examine correlation function $\Phi(\tau;\ 0)$. Frequency shift is feasible due to the instability of frequency of generators, imprecise value of rate of change in the distance between the emitter and the receiver, errors in the tracking of velocity, etc.

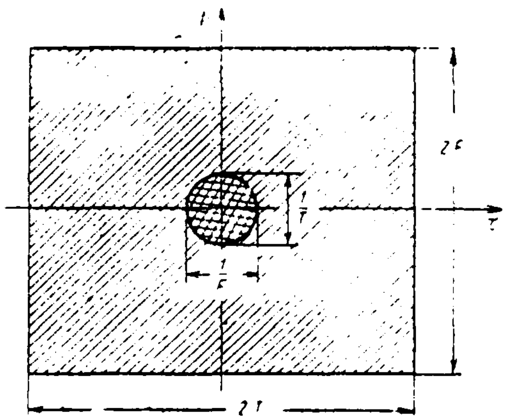

Fig. 2.9. Conditional image of ambiguity function $\Psi(\tau; \Phi)$ of pseudorandom FM signal.

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In the radiolink systems condition (2.172) is satisfied, while in the systems of the evaluation/estimate of the velocity and range this condition must be satisfied at least in one measuring channel. Therefore there is fundamental interest during the study of FM signals in the examination of correlation function $\Psi(\tau)=\Psi(\tau; 0)$.

Many different pseudorandom signals [45, 63], which have favorable correlation functions, are proposed. Among these signals, apparently, greatest propagation find pseudorandom FM signals, comprised on the basis of maximum linear recurrent sequences (Haffmen's signals). In this case should be assigned arbitrary

initial sequence from n symbols $a_1, \ldots, a_n$ (number n - the memory of the code).

Subsequent symbols of code are determined from recursion formula

$$a_i = \mu_0 + \mu_1 a_{i-1} + \ldots + \mu_n a_{i-n}$$
$$(i = n+1, n+2, \ldots). \qquad (2.173)$$

Each of coefficients of code $\mu_0, \mu_1, \ldots, \mu_n$ is also binary number 0 or 1, and addition in (2.173) is conducted on modulus/module 2. Coefficients $\mu_j$ are selected from the code tables of maximum linear recurrent sequences (MLRP) [45]. In this case the sequence is periodic with a length of the period of

$$M = 2^n - 1. \qquad (2.174)$$

Thus, for instance, if we as initial sequence assign 1111 and to use rule of coding for n=4: $a_i = a_{i-4} + a_{i-1}$, then corresponding signal will present sequence from following 15($2^4$-1) elements/cells:

$$1\ 1\ 1\ 1\ 0\ 1\ 0\ 1\ 1\ 0\ 0\ 1\ 0\ 0\ 0. \qquad (2.175)$$

Further continuation of series/row (2.175) brings and to its periodic repetition. Correlation function $\Psi(\tau;\ 0)$ the signal, represented by series/row (2.175), is depicted in Fig. 2.10. Side-lobe level does not exceed 0.25.

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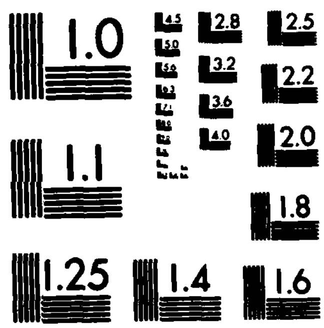


MICROCOPY RESOLUTION TEST CHART
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General law for radar signals of Haffmen lies in the fact that the maximum side-lobe level does not exceed $\frac{1}{\sqrt{M}}$.

. Haffmen's signals can successfully also be utilized in transmission systems of discrete/digital communications/reports, for example in digital telemetry. Let us assume, radio link is intended for the transmission of set/dialing from M of discrete/digital communications/reports $\lambda_1, \ldots, \lambda_M$. Respectively should be provided is shaping M of different signals $s_1, \ldots, s_M$. Let the first signal $s_1$ be assigned by MLRP $a_1, \ldots, a_M$. For example, if M=15, then as this sequence sequence (2.175) can be accepted. Then the subsequent signals can be determined thus. Second signal $s_2 — a_M, a_1, \ldots, a_{M-1}$, third signal $s_3 — a_{M-1}, a_M, a_1, \ldots, a_{M-2}$ and so forth. Each following signal is obtained from preceding/previous by the cyclic permutation of elements/cells. The obtained set/dialing of signals possesses ideal correlation properties [39]

$$\Psi_{ij} = C\int_{-\infty}^{\infty} s_i(t)\, s_j(t)\, dt = \begin{cases} 1 & (1)\ \text{при}\ i = j, \\ -\frac{1}{M} & (1)\ \text{при}\ i \neq j. \end{cases} \qquad (2.176)$$

Key: (1). with.

All signals of set/dialing are equidistant, and with sufficiently large M number it is virtually not correlated or orthogonal with respect to each other.

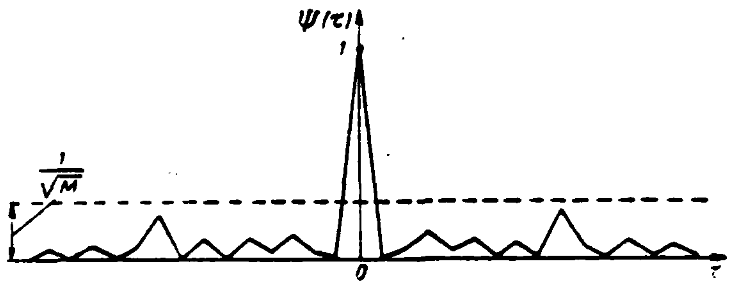


Fig. 2.10. Correlation function of pseudorandom FM signal.

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System of transmission of M discrete/digital communications/reports by means of M orthogonal signals possesses a series/row of advantages in comparison with the usual transmission system of discrete/digital communications/reports to binary, for example, FM, the code. This, first of all, the increased freedom from interference (which will be shown in Chapter 4) and, in the second place, at the same speed of transmission of information - wider frequency spectrum of signal. The latter facilitates moreover in conditions of multiple-pronged propagation and raises the reticence of the work of the system of communications [63].

In the same frequency-time interval, in which is arranged/located set/dialing $s_1, \ldots, s_M$, which consists of M=FT ($F=1/\Delta$) of signals of Haffmen, it is possible to place still M of signals

$s_{M+1}, \ldots, s_{2M}$, opposite to set/dialing $s_1, \ldots, s_M$. If signals $s_{M+1}, \ldots, s_{2M}$ are numbered so that opposite will be signals $s_i$ and $s_{i+M}$, then with i, j=1, ..., M

$$C\int_{-\infty}^{\infty} s_i(t)\, s_{j+M}^{*}(t)\, dt = \begin{cases} -1 \text{ (1) при } i=j, \\ \frac{1}{M} \text{ (1) при } i\neq j. \end{cases} \quad (2.177)$$

Key: (1). with.

Finally, due to rotation of initial phase of oscillations of all signals of set/dialing $s_1, \ldots, s_M$ and set/dialing $s_{M+1}, \ldots, s_{2M}$ on $\pi/2$ it is possible to form still 2M signals $s_{2M+1}, \ldots, s_{3M}$ and $s_{3M+1}, \ldots, s_{4M}$, orthogonal with respect to sets/dialing $s_1, \ldots, s_M$ and $s_{M+1}, \ldots, s_{2M}$. Thus, the total number of signals of Haffmen of orthogonal ones and opposite, which can be placed in the band of frequencies F=1/Δ in time interval of T, is equal 4FT. However, the use/application of opposite and quadrature (out of phase on $\pi/2$) signals requires knowledge or tracking of the initial phase of the oscillations adopted and synchronous heterodyning, which substantially complicates treatment system. During the use of a set/dialing only from M(FT) of signals, for example $s_1, \ldots, s_M$, the initial phase of received signals can be the random immeasurable parameter. Optimum working/treatment in this case is simplified.

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In measuring systems of distance or time lag can be used continuous periodic signals, which consist of repeating with period T of signals of form (2.170). Respectively periodically is repeated sequence $a_1, \ldots, a_M$ [for example (2.175)], that determines signal. Such signals have the ideal correlation function, depicted in Fig. 2.11.

In present paragraph there is interest to examine other ambiguity functions of real radar signals, called frequently in the literature in by packet (of repeating) signals. Complex envelope of coherent radar signal (coherent packet)

$$\dot{S}(t)=\gamma(t)\sum_{i=-\infty}^{\infty}\dot{S}_{e}(t-iT_{e}) \qquad (2.178)$$

is the product of the cutting function $\gamma(t)$ to periodic function $\dot{S}_{\Sigma}(t)$.

$$\dot{S}_{\Sigma}(t)=\sum_{i=-\infty}^{\infty}\dot{S}_{e}(t-iT_{e}). \qquad (2.179)$$

where $\dot{S}_e(t)$ - the envelope of the elementary signal $s_e(t)$, i.e., the signal, undertaken in one repetition period $(0, T_e)$. The elementary signals $s_e(t)$ belong to class examined above of the signals: pulse signals, pulse ChM signals, FM signals, etc.

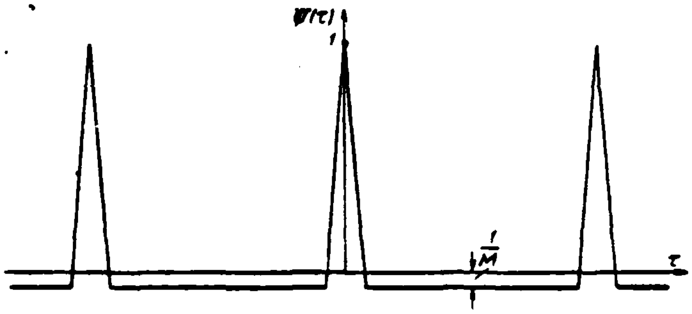


Fig. 2.11. Correlation function of the periodically repeating pseudorandom FM signal.

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Let us record first ambiguity function $\Psi_\Sigma(\tau;\Phi)$ of periodic signal $\dot{S}_\Sigma(t)$. In this case the ambiguity function of the elementary signal $s_0(t)$ we will designate $\Psi_0(\tau;\Phi)$. Taking into account that periodic signal $S_\Sigma(t)$ can be considered as the fold of the elementary signal $\dot{S}_0(t)$ and idealized periodic signal (2.158), on the basis (2.159) and (2.157) we obtain

$$\dot{\Psi}_\Sigma(\tau;\Phi)=C\int_{-\infty}^{\infty}\dot{S}_\Sigma(t)\dot{S}_\Sigma^*(t-\tau)e^{j2\pi\Phi t}dt=$$

$$=C\sum_{i,j=-\infty}^{\infty}\Psi_0(\tau-iT_0;\Phi)\,\delta\left(\Phi-\frac{j}{T}\right). \quad (2.180)$$

Let us further introduce designation for complex ambiguity function of cutting function $\dot{\Psi}_\gamma(\tau;\Phi)$:

$$\dot{\Psi}_\gamma(\tau;\Phi)=C\int_{-\infty}^{\infty}\gamma(t)\gamma^*(t-\tau)e^{j2\pi\Phi t}dt. \quad (2.181)$$

Then after using law (2.156), we find following expression for ambiguity function of real radar signal:

$$\Psi(\tau;\Phi)=C\left|\sum_{i,\,j=-\infty}^{\infty}\dot{\Psi}_{T}\left(\tau;\ \Phi-\frac{j}{T_{0}}\right)\dot{\Psi}_{0}\left(\tau-iT_{0};\ \frac{j}{T_{0}}\right)\right|. \tag{2.182}$$

The effective duration of correlation function $\Psi_T(\tau;0)$

$$T=\int_{-\infty}^{\infty}|\dot{\Psi}_{T}(\tau;0)|^{2}\,d\tau \tag{2.183}$$

in any case several times exceeds the repetition period of the elementary signal $T_0$, i.e., packet contains at least several repeating signals.

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In this case with the high degree of accuracy it is possible to consider that functions $\Psi_T\left(\tau;\ \Phi-\frac{j}{T_0}\right)$ with different values of j do not overlap. Therefore for the operating range $\tau(|\tau|<T_0)$

$$\Psi(\tau;\Phi)=C\sum_{j=-\infty}^{\infty}\left|\dot{\Psi}_{T}\left(\tau;\Phi-\frac{j}{T_{0}}\right)\right|\left|\Psi_{0}\left(\tau;\frac{j}{T_{0}}\right)\right| \tag{2.184}$$

and the fundamental peak of the ambiguity function of coherent

packet, taking into account, that the width of the spectrum of elementary signal is considerably more than the width of the spectrum of the cutting function, it will be determined by the formula

$$\Psi(\tau;\Phi)=|\Psi_{\text{T}}(0;\Phi)||\Psi_{0}(\tau;0)|. \qquad (2.185)$$

Latter means that usually qualitative indices of system on range are determined by form of elementary signal, and on velocity - by form and with duration of cutting function. The ambiguity function of the packet of the repeating impulses/momenta/pulses with the linear law of a change in the frequency is depicted in Fig. 2.12. For the pulse repeating signals without ChM the ambiguity function takes the form, but without the inclination/slope of the principal axes of ellipse with respect to the coordinate axes. The repeating signals are an example of the signals, with which the significant part of the region of high correlation can be extruded/excluded beyond the limits of the working section of plane $\tau$, $\Phi$. Let us emphasize also the possibility of the ambiguity of reading, caused by the fragmentation of the main bang pipe of ambiguity function to large number of the isolated/insulated overshoots.

We examined function of uncertainty/indeterminacy of coherent packet of repeating signals (2.178). The optimum reception/procedure of coherent packet (i.e. the coherent reception/procedure of packet) is feasible in the intervals smaller than the time of the correlation

of the fluctuations of echoing area of target, if the Doppler correction of frequency is known (it is tracked) or there is a diagram of reception/procedure multichannel in the frequency. In the random initial phase of the oscillations of transmitted pulses the memorization and the exception/elimination of this phase, for example, with the aid of the coherent heterodyne is required also.

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In radar of large part is utilized incoherent reception/procedure, with which received signal is received by radio receiving equipment as incoherent packet of repeating signals:

$$\dot{S}(t)=\gamma(t)\sum_{i=-\infty}^{\infty}\dot{S}_{0}(t-iT_{0})e^{j\varphi_{i}}, \qquad (2.186)$$

where $\varphi_i$ - independent random quantities, evenly distributed in interval (0, $2\pi$). The statistical independence of initial phases $\varphi_i$ in different repetition periods can be the result of the rapid fluctuations of the parameters of the channel of communication (in particular, the coefficient of reflection of target), and also the result of failure of the account (measurement) of target speed and initial phase of the oscillations of transmitted pulses.

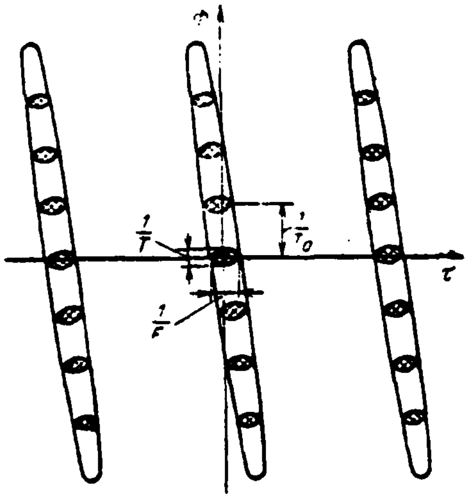


Fig. 2.12. Conditional image of ambiguity function $\Psi(\tau;\ \Phi)$ coherent burst of pulses with linear ChM.

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Ambiguity function is the actually standardized/normalized optimum output effect of radio receiving equipment on the assumption that the interferences at the input are absent. Taking into account the latter and determining on the basis (2.67) the optimum output effect (with $\vec{\lambda}=\tau;\ \Phi$) of radio receiving equipment, we obtain, that the ambiguity function of the incoherent packet of the repeating signals

$$\Psi(\tau;\ \Phi)=\Psi_0(\tau;\ \Phi) \qquad (2.187)$$

coincides with the ambiguity function of the elementary signal $s_0(t)$.

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5.

Potential possibilities of the systems of direction finding according to the method of scanning radiation pattern.

5.1. Ambiguity function of signal in systems of direction finding according to method of scanning radiation pattern.

Let us examine combined ambiguity function of radar signals on range, radial velocity and angle for systems, in which direction finding and permission/resolution in angular coordinate is realized by method of scanning of comparatively sharp radiation pattern. For simplicity of reasonings is examined flat/plane scanning, i.e., the direction finding in one plane. The reading of the angular coordinate of the radiating (or reflecting) object is conducted in the direction of directivity pattern at the moment of time, which corresponds to the passage of the intensity of the oscillations adopted through the

maximum value. Let us designate the complete amplitude-phase radiation pattern of system on the voltage/stress $\gamma(\vartheta)$; angle $\vartheta=0$ corresponds to the direction of maximum amplification, so that $|\gamma(0)|=\max$. The emitted signal $\dot{s}(t)$ is represented in the form of function with the periodically repeating law of modulation $\dot{S}_0(t)$

$$\dot{s}(t)=e^{j2\pi f_0 t}\sum_{i=-\infty}^{\infty}\dot{S}_0(t-iT_0), \qquad (5.1)$$

where $\dot{S}_0(t)$ - complex envelope of elementary signal, $T_0$ - repetition period.

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Further let us assume that emitted signal (5.1) has moderately wide frequency spectrum, in any case of such, with which the antenna radiation pattern for this signal can be considered coinciding with the diagram for the monochromatic signal. Analytically latter/last assumption is reduced to condition $(L/\lambda_0)\leq(f_0/F)$, where L - linear extent of antenna in the appropriate direction, F - width of the spectrum of signal, $f_0$ and $\lambda_0$ - signal carrier frequency and corresponding wavelength. Then the radar signal, reflected from the target with coordinates R, v (respectively $\tau$, $\Phi$) and $\vartheta=\Omega t_0$, where $t_0$ - moment of time, to which the maximum of radiation pattern $\gamma$ is directed toward the target, and $\Omega$ - angular rate of scanning of diagram, it will take the form

$$\dot{s}(t;\ \tau;\ \Phi;\ \vartheta;\ \varepsilon;\ \varphi)=\varepsilon\gamma\left[\Omega(t-t_0)\right]\sum_{i=-\infty}^{\infty}\dot{S}_0(t-\tau-$$
$$-iT_0)\exp\{j\left[2\pi(f_0-\Phi)t+\varphi\right]\}. \qquad (5.2)$$

As earlier, $\epsilon$ and $\varphi$ - parameter of intensity and initial phase, as unessential parameters of signal in question. Let us note that all measured parameters of signal are manpower. In its structure echo signal (5.2) does not differ from the coherent packet of signals (2.178), with exception of the fact that temporary situation $t_0(t_0=\vartheta/\Omega)$ the cutting function depends on angular target position, but the form of the cutting function is determined by the form of radiation pattern. We will consider that the function $\gamma(\vartheta)$ out of the interval $|\vartheta|\leq\pi$ is identically equal to 0.

According to definition, ambiguity function of signal (5.2) is equal to

$$\Psi(\tau_1,\ \tau_2;\ \Phi_1,\ \Phi_2;\ \vartheta_1,\ \vartheta_2)=C\left|\int_{-\infty}^{\infty}\dot{s}(t;\ \tau_1;\ \Phi_1;\ \vartheta_1;\ \varepsilon_1;\ \varphi_1)\times\right.$$
$$\times\dot{s}^*(t;\ \tau_2;\ \Phi_2;\ \vartheta_2;\ \varepsilon_2;\ \varphi_2)\,dt\Big|=C\left|\int_{-\infty}^{\infty}\gamma\left[\Omega(t-t_{01})\right]\gamma^*\left[\Omega(t-\right.\right.$$
$$\left.\left.-t_{01})-\vartheta\right]\dot{S}_\Sigma(t-\tau_1)\dot{S}_\Sigma^*(t-\tau_1-\tau)\,e^{j2\pi\Phi t}dt\right|, \qquad (5.3)$$

where

$$\tau=\tau_2-\tau_1;\ \Phi=\Phi_2-\Phi_1;\ \vartheta=\vartheta_2-\vartheta_1=\Omega(t_{02}-t_{01});$$
$$t_{01}=\frac{\vartheta_1}{\Omega},\quad t_{02}=\frac{\vartheta_2}{\Omega},$$

and C - normalizing factor, such that $\Psi(\tau_1,\ \tau_1,\ \Phi_1,\ \Phi_1,\ \vartheta_1,\ \vartheta_1)=1$.

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If we use definition of

$$\dot{\Psi}_{\gamma}(\vartheta;\ \Phi)=C\int_{-\infty}^{\infty}\gamma(\Omega t)\,\gamma^{*}(\Omega t-\vartheta)\,e^{j2\pi\Phi t}dt \qquad (5.4)$$

for complex ambiguity function of cutting signal $\gamma$ and formula (2.180) for complex ambiguity function $\dot{\Psi}_{\Sigma}(\tau;\ \Phi)$ of periodic signal $\dot{S}_{\Sigma}$ and to interpret (5.3) as Fourier transform product $\gamma\,[\Omega(t-t_{01})]\,\gamma^{*}[\Omega(t-t_{01})-\vartheta]$ and $\dot{S}_{\Sigma}(t-\tau_1)\dot{S}^{*}_{\Sigma}(t-\tau_1-\tau)$, then on the basis of (2.156) instead of (5.3) we will obtain

$$\Psi(\tau_1,\ \tau_2;\ \Phi_1,\ \Phi_2;\ \vartheta_1,\ \vartheta_2)=C\left|\int_{-\infty}^{\infty}\dot{\Psi}_{\gamma}(\vartheta;\ x)\,\dot{\Psi}_{\Sigma}(\tau;\ \Phi-x)\times\right.$$
$$\left.\times\exp\left[j2\pi x\,(t_{01}-\tau_1)\right]dx\right|$$

also, after substitution of (2.180)

$$\Psi(\tau_1,\ \tau_2;\ \Phi_1,\ \Phi_2;\ \vartheta_1,\ \vartheta_2)=C\left|\sum_{i,\ j=-\infty}^{\infty}\dot{\Psi}_{\gamma}\left(\vartheta;\ \Phi-\frac{j}{T_0}\right)\times\right.$$
$$\left.\times\dot{\Psi}_{0}\left(\tau-iT_0;\ \frac{j}{T_0}\right)\exp\left[j2\pi\left(\Phi-\frac{j}{T_0}\right)(t_{01}-\tau_1)\right]\right|. \qquad (5.5)$$

In accordance with actual conditions for work we will consider rate of scanning $\Omega$ of such that duration $T_{\gamma}$ of correlation function $\dot{\Psi}_{\gamma}(\Omega t;\ 0)$ of cutoff signal

$$T_{\gamma}=\int_{-\infty}^{\infty}|\dot{\Psi}_{\gamma}(\Omega t;\ 0)|^{2}\,dt=\frac{1}{\Omega}\int_{-\infty}^{\infty}|\dot{\Psi}_{\gamma}(\vartheta;\ 0)|^{2}\,d\vartheta \qquad (5.6)$$

at least several times exceeds repetition period $T_0$ (approximately/exemplarily $T_T > 5T_0$).

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In this case functions $\Psi_T\left(\vartheta;\ \Phi-\frac{j}{T_0}\right)$ with different values of j, just as function $\Psi_0(\tau-iT_0;\ \Phi)$ with different values of i, virtually do not overlap, and the modulus/module of sum in (5.5) can be replaced with the sum of the moduli/modules of separate components/terms/addends. As a result it is obtained, that the ambiguity function on the range, the velocity and the angular coordinate depends only on a difference in the corresponding arguments that it is characteristic for the signals with the substantially manpower parameters:

$$\begin{aligned}\Psi(\tau_1,\ \tau_2;\ \Phi_1,\ \Phi_2;\ \vartheta_1,\ \vartheta_2) &= \Psi(\tau;\ \Phi;\ \vartheta) = \\ &= C\sum_{i,\ j=-\infty}^{\infty}\left|\dot{\Psi}_T\left(\vartheta;\ \Phi-\frac{j}{T_0}\right)\right|\times \\ &\times\left|\dot{\Psi}_0\left(\tau-iT_0;\ \frac{j}{T_0}\right)\right|. \qquad (5.7)\end{aligned}$$

In operating range of time delays $|\tau|<T_0$ and in interval of frequency shifts $|\Phi|<(1/T_0)$ (5.7) is converted into expression analogous to (2.185):

$$\Psi(\tau;\ \Phi;\ \vartheta) = |\dot{\Psi}_T(\vartheta;\ \Phi)|\,|\dot{\Psi}_0(\tau;\ 0)|. \qquad (5.8)$$

Carried out examination and obtained results of (5.7) and (5.8) make it possible to make a number of practical conclusions, which,

let us recall, are accurate with observance of two conditions:

$$\text{a)}\ \frac{L}{\lambda_0} < \frac{t_0}{F};\quad \text{b)}\ T_\gamma \geq 5T_0. \qquad (5.9)$$

With simultaneous estimation of range, angular coordinate and velocity of radiating (reflecting) object in systems of direction finding according to method of scanning radiation patterns, the potential possibilities of system on range (resolution, accuracy and uniqueness of reading) in this energy relation signal/noise $\mu$ depend only on form of elementary signal, it is more precise from its correlation function $\Psi_0(\tau;\ 0)$.

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Resolution on velocity and accuracy of reading of the parameter $\Phi$ are determined in essence by the duration (of coherent) signal, i.e., with rate of scanning and by width radiation patterns. The potential possibilities of the system on the angular coordinates are determined by the correlation function of the radiation pattern

$$|\Psi_\gamma(\vartheta;\ 0)| = \Psi_\gamma(\vartheta),$$

$$\Psi_\gamma(\vartheta) = C\left|\int_{-\infty}^{\infty}\gamma(\vartheta_1)\,\gamma^*(\vartheta_1-\vartheta)\,d\vartheta_1\right| \qquad (5.10)$$

and on very radiation pattern $\gamma(\vartheta)$ they depend in that measure, in which on it function $\Psi_\gamma(\vartheta)$.

depends.

Potential possibilities of system on angular coordinates do not depend on form of elementary signal. Therefore during further study of these possibilities in present chapter we will consider signal monochromatic which substantially simplifies investigation.

With direction finding according to method of scanning radiation pattern accuracy of reading of angle according to (3.47) is determined by formula

$$\sigma_{\vartheta}^{2}=-\frac{1}{2\mu\frac{d^{2}}{d\vartheta^{2}}\Psi_{T}(\vartheta)|_{\vartheta=0}}, \qquad (5.11)$$

and the angular resolution depends on width of major lobe of function $\Psi_{T}(\vartheta)$ (from extent of region of high correlation), and also on presence and intensity of parasitic lobes of function $\Psi_{T}(\vartheta)$.

During disturbance/breakdown of one of conditions (5.9a) or (5.9b), results of given investigations need refinement. Is of interest the study of the potential possibilities of the systems of direction finding with the antennas of large extent and the broadband signals, for which condition (5.9a) is not satisfied. The short study of this problem will be given in § 7.7.

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§ 5.2. Phenomenon of "compression" of radiation pattern.

Phenomenon in question in present paragraph facilitates increased resolution of systems of lateral survey/coverage with a synthesized aperture. To the description of these systems is dedicated vast, in essence periodic, literature [16, 40, 60, 62, 64, etc.]. We will attempt to present phenomenon with more or less common positions, without being cabled to the concrete/specific/actual systems, since the advisable regions of its use/application are comparatively multi-counted and yet completely determined.

We will consider that being subject to detection and to direction finding radiating or reflecting objects are small size and can be considered as point sources of monochromatic radiation of frequency $f_0$ (wavelength $\lambda_0$). In the general case in the process of the survey/coverage of space and direction finding both the receiving antenna and the radiation source can be moved. For convenience in the presentation let us assume that the radiation source is motionless, and receiving antenna is moved. This assumption does not set any fundamental limitations and it forces only when is moved the radiating object, to consider not the absolute, but relative velocity of receiving antenna.

Let us assume further that in any case in limits of comparatively narrow sector, occupied by main lobe of radiation, antenna system has phase center. The amplitude and phase radiation patterns of antenna system in certain interesting us plane let us designate $\beta(\vartheta)$ and $\chi(\vartheta)$ respectively. So that the specific above complete radiation pattern $\gamma(\vartheta)$:

$$\gamma(\vartheta) = \beta(\vartheta)\exp[j\chi(\vartheta)]. \qquad (5.12)$$

Diagrams $\beta(\vartheta)$ and $\chi(\vartheta)$ are taken by recording of amplitude and phase of oscillation, taken from source of monochromatic radiation, with change in angle $\vartheta$. Angle $\vartheta$ between the direction of the maximum of diagram and the direction to the radiation source subsequently for the brevity is called displacement angle.

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Usually during the removal of radiation patterns a change in the displacement angle $\vartheta$ is conducted by the rotation of antenna system around certain axis. The selection of the method of changing the angle $\vartheta$, in particular the selection of the axis, around which is conducted the rotation of antenna, virtually it is not reflected in the form of amplitude radiation pattern $\beta(\vartheta)$. On the contrary, the

phase radiation pattern of antenna system $\chi(\vartheta)$ depends substantially on the selection of the method of changing the displacement angle $\vartheta$. Thus, for instance, if we turn antenna around the axis, passing through the phase center, the function, which expresses phase radiation pattern, will be the constant number, which can be accepted by the equal to zero.

For determining phase radiation patterns, we will select this law of change in displacement angle, which is caused by working class movement of antenna (or its ray/beam) in process of survey/coverage of space. If under working conditions the survey/coverage of space is conducted with the aid of the rotation of antenna around certain axis, the antenna rigidly connected with the construction/design, then the removal of function $\chi(\vartheta)$ must be produced with a change in the angle $\vartheta$ by the rotation of antenna around this axis. It is analogous, if antenna is established/installed on the flight vehicle and is intended for the direction finding of the landmarks due to scanning of antenna with the displacement of the vehicle, then the removal of the phase radiation pattern must be conducted with the change in the displacement angle $\vartheta$, caused by the working displacement of flight vehicle.

Let us introduce following designations: R - distance between phase center of antenna and radiation source at certain arbitrary

moment of time t, to which corresponds displacement angle $\vartheta$, in general case different from zero; $R_0$ - the same, but at moment of time $t_0$, which corresponds to zero angles of mismatch; $\rho$ - path difference of rays/beams at moment of time t and $t_0$,

$$\rho=\rho(\vartheta)=R-R_0. \quad (5.13)$$

In accordance with determination phase radiation pattern of antenna system is expressed by relation

$$\chi(\vartheta)=\frac{2\pi}{\lambda_0}\rho(\vartheta) \quad (5.14)$$

when displacement angle changes in process of working displacement of antenna during scanning.

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Is of practical interest analytical expression for $\chi(\vartheta)$ at small displacement angles $\vartheta$, not exceeding width $\Delta$ of major lobe of diagram $\gamma(\vartheta)$. In the limits of comparatively narrow major lobe it is possible to approximate phase diagram $\chi(\vartheta)$ by the polynomial of the second power and to disregard/neglect terms of expansion with the degrees of the low parameter $\vartheta$, above second

$$\chi(\vartheta)=\chi(0)+\vartheta\chi'(0)+\frac{1}{2}\vartheta^2\chi''(0)\ (\vartheta<\Delta), \quad (5.15)$$

where

$$\chi'(0)=\frac{d}{d\vartheta}\chi(\vartheta)|_{\vartheta=0};\quad \chi''(0)=\frac{d^2}{d\vartheta^2}\chi(\vartheta)|_{\vartheta=0}. \quad (5.16)$$

In usual antenna systems nonlinearity of phase diagram is expressed weakly, which in effect indicates small change of phase within limits of main lobe caused by nonlinear term in (5.15):

$$\Delta^2\chi''(0)<1. \qquad (5.17)$$

With mutual displacements of oriented emitter (reflector) and antenna, on the other hand, there can occur the relation

$$\Delta^2\chi''(0)\gg 1, \qquad (5.18)$$

which indicates presence of essential nonlinearity of phase diagram.

Respectively in usual systems of survey/coverage of space and direction finding, that use method of scanning highly directional ray/beam, phase radiation pattern in limits of angle, occupied by major lobe, it is possible to consider it as constant $\chi(\vartheta)=\chi(0)$ or changing according to linear law $\chi(\vartheta)=\chi(0)+\vartheta\chi'(0)$. In this case correlation function (5.10) of complete radiation pattern $\gamma(\vartheta)$ is equal to

$$\Psi_{\gamma}(\vartheta)=C\left|\int_{-\infty}^{\infty}\beta(\vartheta_1)\beta(\vartheta_1-\vartheta)\,d\vartheta_1\right| \qquad (5.19)$$

and it is determined only by the amplitude radiation pattern of antenna system.

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The width of correlation function (5.19) or the extent of the region of high correlation on the angular coordinate proves to be to the approximately/exemplarily equal width of radiation pattern Δ. Therefore in the usual systems of the survey/coverage of space and direction finding resolution and accuracy of reading of angular coordinate virtually are determined (with this energy of signal and jamming intensity) the amplitude radiation pattern. in particular by its width, which corresponds to the conventional representations.

When occurs considerable nonlinearity of phase diagram [condition (5.18)], it is possible to substantially increase resolution and accuracy of survey/coverage systems of space and direction finding. In this case correlation function $\Psi_{T}(\vartheta)$, which determines the potential angular resolution and accuracy, takes the form

$$\Psi_{T}(\vartheta)=C\left|\int_{-\infty}^{\infty}\beta(\vartheta_{1})\exp\left[j\frac{\vartheta_{1}^{2}}{2}\chi''(0)\right]\beta(\vartheta_{1}-\vartheta)\exp\times\right.$$
$$\left.\times\left[-j\frac{(\vartheta_{1}-\vartheta)^{2}}{2}\chi''(0)\right]d\vartheta_{1}\right|=C\left|\int_{-\infty}^{\infty}\beta(\vartheta_{1})\beta(\vartheta_{1}-\vartheta)\exp\times\right.$$
$$\left.\times[j\vartheta\chi''(0)\vartheta_{1}]\,d\vartheta_{1}\right| \qquad (5.20)$$

and is obtained substantially narrower than the width of the radiation pattern Δ. As a result it proves to be possible to produce the permission/resolution of radiation sources, which are located in range of the main lobe of radiation. Interconnection it is here accurately the same as between complex signal amplitude envelope $\dot{S}(t)$

and its correlation function $\Psi(\tau)$. When radiation patterns $\gamma(\vartheta)$ it is possible to consider as the real function of angle, correlation function $\Psi_\gamma(\vartheta)$. which determines the potential possibilities of system, differs little from amplitude of the pattern $\beta(\vartheta)$. If the radiation pattern $\gamma(\vartheta)$ is complex with the essential nonlinearity of argument (that equivalent to the frequency modulation of signal $\dot{S}(t)$), correlation function can considerably differ from $\beta(\vartheta)$.

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Let us give the quantitative estimation of this phenomenon.

Amplitude radiation pattern $\beta(\vartheta)$ antenna system we will approximate, as this frequently is done in theoretical studies, bell (Gaussian) curve

$$\beta(\vartheta)=\beta_0\exp\left(-\frac{\pi\vartheta^2}{2\Delta^2}\right), \qquad (5.21)$$

where $\Delta$ - effective width of radiation pattern, which differs little in the case of bell curve from width, counted off at the level of half power. For the usual systems, which satisfy condition (5.17), the correlation function of complete radiation pattern according to (5.19) is equal to

$$\Psi_\gamma(\vartheta)=\exp\left(-\frac{\pi\vartheta^2}{4\Delta^2}\right). \qquad (5.22)$$

In this case effective width of correlation function $\sqrt{2}\Delta$

actually/really differs little from width of radiation pattern. The potential accuracy of reading of the angles of coordinate, determined by formula (5.11)

$$\sigma_{\phi}^{2}=\frac{\Delta^{2}}{\pi\mu},\qquad(5.23)$$

also wholly depends (with this value $\mu$) on the width of diagram $\Delta$.

For systems with essential nonlinearity of phase diagram, introducing designation

$$\frac{1}{2\pi}\chi''(0)\Delta^{2}=d,\qquad(5.24)$$

after substitution (5.21) in (5.20) we obtain

$$\Psi_{\tau}(\vartheta)=\exp\left[-\frac{\pi\vartheta^{2}}{4\Delta^{2}}(1+4d^{2})\right].\qquad(5.25)$$

comparison (5.25) with (5.22) shows that with essential nonlinearity of phase radiation pattern, expressed by condition (5.18), correlation function $\Psi_{\tau}(\vartheta)$, retaining the same form of bell curve, becomes narrow $\sqrt{1+4d^{2}}\approx 2d$ once.

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As a result in so many once grows/rises the resolution and accuracy on the angular coordinate. Value 2d subsequently is called contraction coefficient. Contraction coefficient, as this follows from (5.24) is determined by the characteristics of antenna and with an accuracy to constant factor $1/2\pi$ is equal to the product of the

squares of the width of radiation pattern to the second derivative of phase diagram $\chi''(0)$.

In certain cases during investigation of survey/coverage systems of space and direction finding it is convenient to extract directly signals, induced in antenna by radiation sources, and to examine possibility of permission/resolution of these signals without determination of phase radiation pattern. Let us assume in the space, examined/scanned by antenna during the scanning, there is point source of monochromatic radiation $S_0 \exp(j2\pi f_0 t)$ in the direction of $\vartheta_0$, which corresponds to time $t_0$. If $\vartheta(t-t_0)$ and $R(t-t_0)$ - the laws of a change in the displacement angle and distance between the phase center of receiving antenna and the radiating object in the process of their mutual displacement, then the signal, induced in the antenna, taking into account the random ones of intensity $\epsilon$ and initial phase $\varphi$ will be expressed by the formula

$$\dot{s}(t;\ \vartheta_0;\ \varepsilon;\ \varphi) = \varepsilon S_0 \beta\,[\vartheta(t-t_0)] \times$$
$$\times \exp\left\{j\left[2\pi f_0 t + \frac{2\pi}{\lambda_0} R(t-t_0) + \varphi\right]\right\}. \quad (5.26)$$

Instead of distance of R in (5.26) can figure path difference $\rho = R - R_0$. The signals, reflected from the objects, which have the identical laws of a change in distance of R(t), differ from each other only in terms of the value of time lag $t_0$ (by angular position) and in terms of the unessential parameters $\epsilon$ and $\varphi$. Respectively, the

possibility of the permission/resolution of the targets, which have different angular coordinates $\vartheta_{01}$ and $\vartheta_{02}$, is reduced to the possibility of the permission/resolution of identical (with an accuracy to phase and intensity) signals (5.26), which have different time lag $t_{01}$ and $t_{02}$. If distance R between the object and the phase center of antenna constantly or changes linearly, then signal has a character of radio pulse with the constant carrier frequency.

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For the resolution of such signals they must in the time virtually not overlap. On the contrary, when during the determination of emitter in range of the main lobe of radiation distance R changes considerably and it is nonlinear, received signal (5.26) is the radio pulse, the carrier which is frequency-modulated. In particular, under the quadratic law of a change in distance

$$R(t-t_0)=R(0)+(t-t_0)R'(0)+\frac{1}{2}(t-t_0)^2R''(0) \quad (5.27)$$

frequency modulation is linear with a speed of change in frequency of df/dt, determined by the second derivative of argument (Arg) of complex signal (5.26)

$$\frac{df}{dt}=\frac{1}{2\pi}\frac{d^2}{dt^2}\operatorname{Arg}[\dot{s}(t;\ \vartheta_0;\ \varepsilon;\ \varphi)]=\frac{R''(0)}{\lambda_0}. \quad (5.28)$$

Taking into account that rate of change in frequency is connected with total variation in frequency $2F_m$ for time of effective

duration of signal T with relation

$$\frac{df}{dt}=\frac{2F_m}{T}, \qquad (5.29)$$

signal (5.26) under quadratic law of change in distance of R can be also expressed through usually utilized parameters of signals, linearly frequency-modulated.

Thus, under rapid and nonlinear law of change in distance signal, accepted from monochromatic radiation source, proves to be frequency-modulated that it causes expansion of pulse spectrum and provides possibility of its temporary/time compression, using known methods of optimum filtration of frequency modulated impulses/momenta/pulses. As a result proves to be possible to produce the permission/resolution of the overlapping signal pulses, i.e., the permission/resolution of sources, which are located in range of the main lobe of radiation.

Both given treatments of phenomenon of "compression" of the radiation pattern by means of investigation of the phase diagram (5.15) and by means of investigation of signals (5.26), induced in antenna, they are identical.

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Difference is reduced to the fact that in the examination of phase

diagram distance R or path difference $\rho$ represent in the form of the function of the angle of mismatch, whereas in (5.26) - in the form of the function of time. The identity of treatments is complete, when displacement angle $\vartheta$ in the limits of major lobe can be considered the linear function of time. The advisability of using one or the other treatment in the specific cases is a question of convenience.

Examination given in present paragraph makes it possible to make following conclusion. When during the determination of the oriented radiating (reflecting) object within the limits of the alignment of the main lobe of radiation an essential and nonlinear change in the distance between the object and the phase center of antenna occurs, is possible the realization of the "compression" of radiation pattern, which is evinced by the considerable contraction of correlation function $\Psi_T(\theta)$ of complete radiation pattern in comparison with the width of amplitude diagram $\beta(\vartheta)$, which substantially increases resolution and accuracy. The practical realization of the phenomenon of "compression" is reduced to the optimum filtration or the correlation reception of coherent signal in the time interval, equal to the retention time of the oriented object in range of the main lobe of radiation. For this the observance of the following conditions is required. First, it is necessary to know previously the law of a change in distance of $R(t-t_0)$ between the phase center of antenna and the oriented object or the ensemble of

the laws of a change in distances of $R(t-t_0; \nu)$, depending on certain parameter $\nu$, which determines, for example, range at the moment of time $t=t_0$, the second angular coordinate of object, etc. In the second place, the signal, emitted by the oriented object, must have a character of the determined function, known with an accuracy to the unessential parameters of intensity $\epsilon$ and initial phase $\varphi$ at the values of the measured parameters given previously.

Let us illustrate aforesaid based on several practical examples. For simplification in the study of phenomena we will further assume that the emitted signal is monochromatic, although the practical realization of the system (but not its potential possibilities on the angular coordinate) it depends on the form of the emitted signal.

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5.3. Examples of the practical use of a phenomenon of the "compression" of radiation pattern.

Let us assume, antenna is established/installed on board aircraft and scanning with survey/coverage of Earth is realized due to displacement of aircraft [16]. If we consider that the aircraft (antenna) is moved rectilinearly and evenly at a rate of v and to designate by letter $\alpha$ the angle between vectors of speed and

direction to the radiating object at the moment of time $t_0$ (when displacement angle is equal to zero), then from the simple geometric correlations we find:

$$\frac{d}{dt}R(t)\Big|_{t=t_0}=v\cos\alpha,$$

$$\frac{d^2}{dt^2}R(t)\Big|_{t=t_0}=\frac{v^2}{R_0}\sin^2\alpha, \qquad (5.30)$$

$$\frac{d^3}{dt^3}R(t)\Big|_{t=t_0}=3\frac{v^3}{R_0^2}\sin^2\alpha\cos\alpha.$$

Most favorable for using effect of "compression" of the radiation pattern is case, when direction of maximum antenna gain lies/rests at plane, perpendicular to velocity vector:

$$\alpha=\frac{\pi}{2}.$$

In this case the expansion of function R(t) or $\rho(t)$ in the vicinity of point $t=t_0$ according to the degrees of interval $(t-t_0)$, which does not exceed the retention time of emitter in range of the narrow main lobe of radiation, will contain virtually only quadratic term. Correspondingly, in the signals induced in the antenna by the sources of monochromatic radiation, the Doppler shift of the carrier frequency will be absent, and frequency modulation will be linear.

Let us examine this case, by assuming that velocity vector of aircraft lies/rests at horizontal plane. Situation is represented in Fig. 5.1: $\Phi_0$ and $\Phi$ - positions of the phase center of antenna at the moments of time $t_0$ and t respectively; $\vartheta$ - displacement angle at the

arbitrary moment of time t; И - point radiation source; $R_0$ - distance between the phase center $\Phi_0$ and the radiation source И at the moment of time $t_0\left(R_0=\frac{H}{\cos\beta}\right)$; $H$- flight altitude; $\beta$ - angle between the direction of sighting and the vertical line at the moment of time $t=t_0$.

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From the figure it follows that the displacement angle

$$\vartheta=\operatorname{arc\,tg}\frac{v(t-t_0)}{R_0}\approx\frac{v(t-t_0)}{R_0}\quad \text{(with } \vartheta\ll 1\text{)},$$

i.e. changes linearly with a constant angular velocity of $\Omega=v/R_0$. The distance between the radiation source И and the phase center Φ at the moment of time t

$$R=R_0+\rho=\frac{R_0}{\cos\vartheta}=R_0\sqrt{1+\operatorname{tg}^2\vartheta}=$$
$$=R_0\sqrt{1+\left[(t-t_0)\frac{v}{R_0}\right]^2}, \qquad (5.32)$$

whence we obtain a path difference

$$\rho=\frac{R_0}{2}\frac{(t-t_0)^2v^2}{R_0^2}-\frac{R_0}{8}\frac{(t-t_0)^4v^4}{R_0^4}+\ldots\approx$$
$$\approx\frac{(t-t_0)^2v^2}{2R_0}, \qquad (5.33)$$

phase radiation pattern

$$\chi(\vartheta)=\frac{2\pi}{\lambda_0}\rho=\frac{\pi R_0\vartheta^2}{\lambda_0} \qquad (5.34)$$

and the coefficient of "compression"

$$2d=\frac{1}{\pi}\chi''(0)\Delta^2=\frac{2R_0\Delta^2}{\lambda_0}. \qquad (5.35)$$

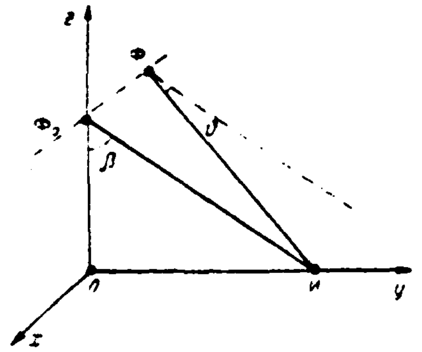


Fig. 5.1. Lateral survey/coverage of the surface of the Earth from the aircraft $ИΦ_e = R_e$; $ИΦ = R_e + \rho$; $OΦ_e = H = R_e \cos\beta$.

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All parameters in question - width of radiation pattern, phase radiation pattern, resolution - relate to plane, passing through velocity vector and line of sighting. The angular resolution, evaluated by the width of the region of high correlation $\vartheta_{кор}$, is equal to

$$\vartheta_{кор} = \frac{\sqrt{2}\,\Delta}{2d} \approx \frac{\lambda_e}{R_e \Delta} \approx \frac{L}{R_e}. \qquad (5.36)$$

In (5.36) it is accepted that width of diagram $\Delta = \lambda_e / L$, where L - linear extent (length) of antenna in appropriate direction. The linear resolution of system along the line of the flight

$$l_{кор} \approx \vartheta_{кор} R_e = L. \qquad (5.37)$$

Let us discuss obtained results. According to (5.36) and (5.37), the angular resolution the higher ( $\theta_{кор}$ the less), the less the length of antenna and the wider its radiation pattern, and the linea- resolution does not depend on distance and is equal to the length of antenna. Furthermore, the resolution of system $(\theta_{кор}, l_{кор})$ does not depend on flight speed v. An increase in the resolution in proportion to decrease L is explained by the fact that with the decrease of the length of antenna increases the width of its radiation pattern and, consequently, also the exposure time of target (duration of signal T). At the given rate of change in the frequency the width of the spectrum of received signal $2F_m$ grows/rises and it becomes possible to solve signals with the smaller temporary/time (angular) shift/shear. However, this law is accurate only as long as the main lobe of radiation remains sufficiently to narrow ones. During further expansion of diagram it is necessary to consider the highest terms of the expansion of series (5.33) which leads to the nonlinear frequency modulation of received signals and the disturbance/breakdown of the obtained laws. Furthermore, increases the effective duration of signal T:

$$T = \frac{\Delta}{\Omega} = \frac{R_0 \Delta}{v}, \qquad (5.38)$$

whereas the optimum processing of signals and their effective compression are possible only in comparatively small time intervals (order of several tenths of second and less), in which there can be ensured the coherence of signal. Let us illustrate the importance of

last consideration based on practical example. If we accept $R_0=10^4$ m, $\lambda_0=3\cdot10^{-2}$ m, $\Delta=0.017$ rad ($\approx1°$), $v=300$ m/s, then $2d\approx400$ and $T=0.5$ s. Consequently, even with a comparatively narrow radiation pattern the duration of signal from the point of view of the possibility of its coherent processing has critical value. For the same reasons the velocity of the displacement of antenna v has a value. As a result for the given specific conditions for the work of system the optimum sizes/dimensions of antenna or the optimum width of radiation pattern can be determined.

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In the literature it is emphasized that phenomenon of "compression" of diagram is not in contradiction with conventional representations, according to which maximum resolution, determined by minimum width of the radiation patter, is approximately equal to $\lambda_0/L$. Actually/really, moving antenna accepts radiation/emission from the oriented object, until the latter is located in range of major lobe of the radiation pattern. For this time the antenna passes path of $R_0\Delta$ and forms the imaginary antenna with an equivalent length of $L_{экв}=R_0\Delta$. The width of the radiation pattern of the imaginary or equivalent antenna can be $\frac{\lambda_0}{L_{экв}}$ (i.e. $\lambda_0/R_0\Delta$), which coincides with (5.36).

Let us pay attention to one more fact, which was not considered in given examination. Phase radiation pattern (5.34) for the emitters, which are found on different ones of distance $R_0$, is different. With respect to different there will be for the signals, taken from these emitters, rate of change in the frequency. The permission/resolution of the sources of monochromatic radiation in the range makes the latter in the principle with possible [16]. However, range resolution is comparatively low and for its increase apply broadband, in particular pulse, radiation/emission.

Relationships analogous to those which were obtained in examination of direction finding from aircraft, occur with survey/coverage of earth's surface from satellites [16], derived in circular orbit (Fig. 5.2). In the figure, as earlier, $\Phi_0$ and $\Phi$ are two positions of the phase center of antenna at the moments of time $t_0$ and $t$, respectively; И - radiating point; $\vartheta$ - displacement angle (it is assumed that the direction of the maximum of radiation pattern is oriented along the normal to the trajectory of satellite); H - height/altitude of circular orbit; r - radius of the earth. Furthermore, one should consider that the satellite, derived in circular orbit, has the constant velocity, equal to

$$v = 7{,}91\sqrt{\frac{r}{r+H}}\ \text{km/s}, \qquad (5.39)$$

and the constant angular velocity

$$\Omega = \frac{v}{r+H}. \tag{5.40}$$

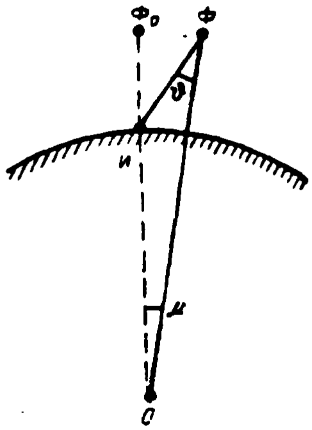


Fig. 5.2.

Fig. 5.2. Survey/coverage of the Earth with ISZ [ИСЗ - artificial earth satellite]. $ИФ_0$=H; ОИ=r; ИФ=H+$\rho$; ОФ=H+r.

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From the figure it follows that.

$$\frac{\sin\vartheta}{r}=\frac{\sin(\vartheta+\mu)}{r+H}. \qquad (5.41)$$

Taking into account smallness of angles $\mu$ and $\vartheta$ latter/last relationship/ratio is converted in

$$\vartheta=\mu\frac{r}{H}, \qquad (5.42)$$

and, therefore, displacement angle is linear function of time

$$\vartheta=\frac{r}{r+H}\frac{(t-t_0)\,\sigma}{H}. \qquad (5.43)$$

We further find distance ИФ (H+$\rho$):

$$H+\rho=\sqrt{r^2+2r(r+H)\cos\mu+(r+H)^2}\approx$$
$$\approx H\sqrt{1+\frac{r+H}{r}\vartheta^2}. \qquad (5.44)$$

In all in practice interesting cases second term in radicand is substantially lower than one. Therefore

$$\rho\approx\frac{r+H}{r}\frac{H\vartheta^2}{2}. \qquad (5.45)$$

We respectively obtain: phase radiation pattern

$$\chi(\theta)=\frac{r+H}{r}\frac{\pi H\theta^2}{\lambda_0}, \qquad (5.46)$$

contraction coefficient

$$2d=\frac{r+H}{r}\frac{2H\Delta^2}{\lambda_0} \qquad (5.47)$$

and the effective duration of the received signals

$$T=\frac{H\Delta}{v}\frac{r+H}{r}. \qquad (5.48)$$

Usually H<<r and ratio (r+H/r) in given formulas can be accepted by equal to one. If we, as in the preceding case, place $\lambda_0=3\cdot10^{-2}$ m, $\Delta$=0.017 rad, and trajectory height of satellite to take as equal to $H=2\cdot10^5$ m (corresponding value of the satellite velocity v=7.90 km/s), then there will be obtained $2d=7.5\cdot10^3$ and T=0.43 s.

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During conclusion/output of relationships/ratios, connected with survey/coverage of Earth from satellite (Fig. 5.2), maximum of antenna radiation was oriented on vertical line down. If the maximum of radiation/emission is oriented again in the plane, normal to the flight trajectory, but at certain angle $\beta$ to the vertical line, then are obtained the analogous relationships in which only H should replaced by $R_0=H/\cos\beta$.

In examined examples the basic is the change in distance between

phase center of antenna and oriented radiation source is. In this case it is not important, which of two interacting objects (antenna or emitter) remains motionless, and what it is moved. Therefore, in particular, the results, obtained in a latter/last example, can be used to the turned case - to the case of trajectory measurements [17], when the antenna, arranged/located on the Earth, is motionless, and the satellite or radiation source, which is located on the satellite, is oriented. Let us assume, the direction $\vartheta=0$ of the maximum of the radiation pattern of the motionless antenna, arranged/located on the Earth, is oriented on the vertical line upward. The oriented satellite is found in circular orbit with a known height/altitude of H (Fig. 5.3). in the figure Φ - the phase center of motionless antenna; $И_0$ and И - positions of the center of mass of satellite (emitter) at the moments of time $t_0$ and t respectively. It is necessary with the maximum accuracy to rate/estimate the moment/torque of intersection by the satellite of plane, normal line of flight and by passing through the direction $\vartheta=0$ (moment/torque of passage by the satellite of the crosspiece of antenna). The law of a change of the path difference in this case, as in preceding/previous, it is described by formula (5.45). Respectively phase radiation pattern and contraction coefficient of diagram are again expressed by relationships/ratios (5.46) and (5.47).

Latter/last example is interesting to those that illustrates possibility with motionless antenna (and moving oriented object) to have to effective antenna (and moving oriented object) to have effective or "compressed" width of radiation pattern, considerably smaller than ratio of wavelength $\lambda_0$ to extent of aperture L, and to provide respectively high angular resolution and accuracy. Treatment about the imaginary antenna here is not applicable. Antenna accepts radiation/emission from the oriented object in the space, whose extent is equal to the length of the real aperture of antenna. Consequently, the resolving is not the extent of the space, occupied by antenna in the process of its interaction with the oriented object, but the qualitative change in the phase antenna radiation pattern or phase of received signal, caused by the mutual displacement of antenna and oriented unit.

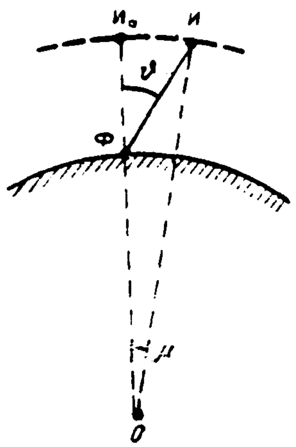


Fig. 5.3. Direction finding with the aid of the motionless antenna of the source, arranged/located on ISZ, $ФИ_0=H$; $ОФ=r$; $ФИ=H+\rho$; $ОИ_0=r+H$.

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Noted possibility of using phenomenon of "compression" of radiation pattern or principles, placed as basis of action of antennas with synthesized aperture, in systems of trajectory measurements of orbital objects is of great practical interest. In particular, is of interest the indicated in [67] identity according to qualitative indices of the systems of the optimum filtration of the signals, taken by synthesized aperture, and the measuring systems of Doppler frequency. Signal frequency, taken from the source, which is located on the satellite, which for simplicity relies by that derived on the circular orbit (Fig. 5.3), according to (5.43) and (5.46) is expressed by the formula

$$f=f_0+\frac{d}{dt}\frac{\chi(\theta)}{2\pi}=f_0+\frac{v}{\lambda_0}\theta$$

and is determined the angular position $\vartheta$ of satellite relative to the orienting station. Analogous results and corresponding formulas can be obtained in the more general case, when satellite injected into an elliptical orbit, and station Φ is located out of the orbital plane. Is necessary only in accordance with what has been said above the preliminary knowledge of orbit or ensemble of the possible orbits of satellite.

Let us give one additional example to possibility of using phenomenon of "compression" of radiation pattern [62]. We assume (for example, in early-warning radar), that there is conducted flat/plane rotation or oscillation of comparatively narrow radiation pattern around point O, which does not coincide with the phase center of antenna (Fig. 5.4). On the figure there are depicted radiating (reflecting) point И, two positions of phase center $\Phi_0$ and Φ at the moments of time $t_0$ and t, r - radius of gyration of phase center, $R_0$ and $R_0+\rho$ - distance of the radiating object at the moments of time $t_0$ and t respectively. The direction of the maximum of radiation pattern is oriented along the normal to the trajectory, described by phase center. From the figure it follows that

$$\frac{\sin\mu}{r}=\frac{\sin\theta}{R_0+r}=\frac{\sin\nu}{R_0+\rho}. \qquad (5.49)$$

Keeping in mind smallness of angles $\mu$, $\vartheta$, $\nu$ and condition $R_0 \gg r$, it can be assumed that

$$\mu = 0; \ \vartheta = \nu = (t - t_0)\Omega. \qquad (5.50)$$

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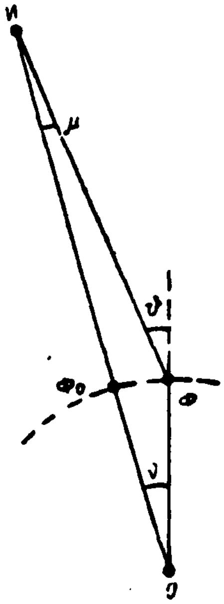


Fig. 5.4. Direction finding with the use of rotation of antenna around the axis not passing through the phase center.
$ОΦ=ОΦ_0=r(r+H)$; $ИΦ_0=R_0$; $ИΦ=R_0+\rho$.

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Further,

$$R_0+\rho=\sqrt{(R_0+r)^2+r^2-2(R_0+r)r\cos\vartheta}\approx$$
$$\approx R_0+r(1-\cos\vartheta), \quad (5.51)$$

whence

$$\rho=\frac{r\vartheta^2}{2}, \quad (5.52)$$

and, therefore, phase radiation pattern and compression coefficient are equal to

$$\chi(\vartheta)=\frac{\pi r\vartheta^2}{\lambda_0} \quad (5.53)$$

and

$$2d=\frac{2r\Delta^2}{\lambda_0}. \quad (5.54)$$

The gain in angular resolution and accuracy in early-warning radar described by us is caused by motion of antenna in circle/circumference of comparatively large radius. Respectively grow/rise the sizes/dimensions of the space, occupied by the antenna in the process direction finding. It is natural that the antenna, which occupies all this space (i.e. the increased sizes/dimensions) and which rotates around the axis, which passes near the phase center, will also have an increased resolution and an accuracy. However, it is possible to visualize the cases, when is more profitable to have the small antenna, moving in the circle/circumference, than the large antenna, which rotates around its axis. In all remaining given examples the mutual displacement of antenna and oriented unit is organically inherent in systems themselves, ane they are not created artificially. In this case the "compression" of radiation pattern and an increase in the resolution and accuracy are obtained only after it on account of the use of optimum coherent perfecting of the oscillations adopted.

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List of principal notations.

List of principal notations, used in text, is given below. Many designations are encountered for the elongation/extent of the entire book, whereas others have the limited use. In the latter case the chapters, where these designations are used, are indicated. Chapters are indicated also for those designations, which make different sense in different chapters.

$A=\|A_{ij}\|$, $B=\|B_{ij}\|$ — informational matrices/dies when is fixed/recorded the oscillation/vibration adopted or the transmitted communication/report respectively.

C - constant coefficient, normalizing factor.

c - velocity of propagation of electromagnetic waves.

d - product of the width of the spectrum of signal to its duration.

D - distance between the phase centers of two antennas.

E - energy of signal.

f - frequency.

$f_0$ - carrier frequency.

F - effective width of the spectrum of signal.

$F_{ck}$ — root-mean-square width of the spectrum of signal.

F(t) - the law of frequency modulation.

F - rate of change in the frequency.

$\mathcal{F}[\ ]$ and $\mathrm{arc}\,\mathcal{F}[\ ]$ — straight/direct and inverse transformation of Fourier.

G(f) - the spectrum of complex signal amplitude envelope S(t).

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h(t), $h(t_1;\ t_2)$ - the pulse responses of linear system.

i, j, k, l, m, n - natural numbers.

I(z) - amplitude-phase distribution on antenna aperture (chapter 7).

k - constant coefficient.

$l$ - likelihood ratio (Chapter 1), the symbol of the envelope of high-frequency process.

L - extent of the aperture of line-source antenna.

$\mathcal{L}(\vec{\mathcal{L}})$ - one-dimensional (multidimensional) region, abstracted/removed under the aperture of antenna system.

m - number of solvable elements/cells in the space of communications/reports, a number of channels in the diagram of optimum working/treatment, numerical length in the binary communication/report (chapter 4).

M - number of transmitted communications/reports, a number of levels of quantization, a number of elements/cells in the pseudorandom FM signal.

n(t), $n(t; \vec{r})$ - the random function, which presents interferences.

$\dot{n}(t)$, $\dot{N}(t)$ - complex representation and complex envelope of interference n(t).

N, N* - the numbers, transmitted in the digital communication system, and their evaluation/estimate.

$\mathcal{N}_0$ — spectral intensity of the field of interferences at the input of receiving antenna [$V^2$ s/m].

p(λ) - the probability density of continuous random variable λ.

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$P(\lambda_i)$ — distribution of probability of discrete random variable $\lambda_i$.

$P_{\text{om}i}(P_{\text{om}})$ — conditional probability of error during the transmission of the i signal.

$P_{\text{om.cp}}$ — average/mean probability of errors during the data transmission ensemble of communications/reports.

p(x|y) - conditional probability density x under the condition of

realization y.

$p(\lambda|\vec{u})$ - a posteriori probability density.

$p(\vec{u}|\lambda)$ - the function of plausibility.

$q(\lambda)$ - correlation integral.

$q_{\perp}(\lambda)$ - correlation integral for the conjugated/combined signal $s_{\perp}$.

$Q(\lambda)$, $Q(\lambda)$ - complex correlation integral and its modular value.

$\dot{Q}(\lambda)$, $Q(\lambda)$ — signal function.

$q_s(\lambda_1; \lambda_2)$ — signal function for the conjugated/combined signal $s_{\perp}$.

$q_{s\perp}(\lambda_1; \lambda_2)$ — complex signal function and its modular value.

$\vec{r}$, r - radius-vector and its modulus/module.

$R(t_1; t_2)$, $R(t_1, t_2; \vec{r}_1, r_2)$ - correlation functions. In the combined examination of several random processes of $n(t)$, $\epsilon(t)$ and so forth; correlation functions they are designated $R_n$, $R_\epsilon$ and so forth.

R - distance from the phase center to the oriented radiation source (Chapter 5).

$s(t; \lambda)$ - the transmitted (or adopted) radio signal.

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$s_1(t; \lambda)$ - the function, conjugated/combined according to Gilbert/Hilbert (or quadrature) to $s(t; \lambda)$.

$\dot{s}(t; \lambda)$, $\dot{S}(t; \lambda)$ - complex representation and complex signal amplitude envelope $s(t; \lambda)$.

$s_0(t; \lambda)$, $S_0(t; \lambda)$ - elementary signal and its complex enveloping.

$s_\Sigma(t; \lambda), \dot{S}_\Sigma(t; \lambda)$ — periodic sequence of elementary signals and its complex enveloping.

t - time.

$T_0$ - repetition period.

T - effective duration of signal, the duration of the interval of observation (Chapter 8).

$T_{cк}$ — root-mean-square duration of signal.

$(T_1, T_2)$ - the interval of observation.

$u(t)$, $u(t; \vec{r})$ - the oscillation/vibration adopted.

$\dot{u}(t)$, $\dot{U}(t)$ - complex representation and complex envelope of oscillation/vibration $u(t)$.

v - rate (Chapter 5).

$v(\lambda_m)$ — vicinity of point $\lambda_m$.

$v_t(t_1; t_2)$ — nucleus, reverse/inverse to $\rho_t(t_1; t_2)$.

x, y, z(r, $\beta$, $\vartheta$) - space coordinates in the Cartesian (spherical) coordinate system.

$Y(\lambda)$, $Z(\lambda)$ - the output effects of the system of processing signals.

$\alpha(\vec{\alpha})$ - the scalar (vector) immeasurable parameter of signal.

$\beta$ - angular coordinate in the spherical coordinates.

$\beta(\vartheta)$ - the amplitude antenna radiation pattern.

$\gamma(\vartheta)$, $\gamma(\theta)$ - the complete (complex) radiation pattern of antenna system.

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$\gamma(t)$ - the cutting function.

$\Gamma(f)$ - the Fourier transform of the cutting function.

$\delta(t)$, $\delta_{ij}$ — delta function and the symbol of Kronecker.

$\Delta$ - space of quantization, the duration of the elementary pulses of pseudorandom FM signal, the effective width of the antenna radiation pattern (Chapter 6, 8).

$\varepsilon$, $\varepsilon$ - complex parameter of intensity $(\varepsilon=\varepsilon e^{j\varphi}=\varepsilon_c+j\varepsilon_s)$ and its modulus/module.

$\varepsilon_0$ — mathematical expectation $\varepsilon(\langle\varepsilon\rangle=\langle\varepsilon_0\,\varepsilon_c\rangle=\varepsilon_0,\ \langle\varepsilon_s\rangle=0)$.

$\vec{\eta}(\vec{\eta}_*)$ — unit vector, arranged/located in the direction $\beta$, $\vartheta$.

$\eta_i(t)$ — output effect of the i servo meter (Chapter 6).

$\vartheta$ - measured angular coordinate, angular coordinate in the spherical coordinates.

$\theta=\sin\vartheta$ - generalized measured angular coordinate.

$\Theta(t_1, t_2)$, $\Theta(t_1, t_2, \vec{r}_1, \vec{r}_2)$ - the nucleus, reverse/inverse of the correlation function of the oscillations/vibrations u(t) and u(t adopted; $\vec{r}$) respectively.

$\dot{\theta}$ - rate of change $\theta$.

$\lambda(\vec{\lambda})$ - the scalar (vector) measured parameter of signal, wavelength (Chapter 5, 7, 8).

$\lambda^*$, , $\vec{\lambda}^*$ - evaluation/estimate of the parameters $\lambda$ and $\vec{\lambda}$, maximum likelihood estimate (Chapter 6).

$\lambda^{**}$, $\vec{\lambda}^{**}$ - evaluation/estimate according to the maximum of a posteriori probability (Chapter 6).

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$\lambda_{и}, \vec{\lambda}_{и}$ — true value of the measured parameter.

$\lambda_m, \vec{\lambda}_m$ — point of the maximum of the function of plausibility.

$\lambda_0, \vec{\lambda}_0$ — reference value of the measured parameter.

$\lambda_{iкор}$ — width of the region of high correlation from parameter.

$\lambda_{мин}, \lambda_{макс}$ — range of change in the parameter $\lambda$.

$\Lambda$, $\Lambda^*$ - space of communications/reports and the space of evaluations/estimates.

$\mu$ - energy relation is signal/noise.

$\nu_i(t)$ — error of the i servo meter (Chapter 6).

$\rho$ - path difference of rays/beams (Chapter 5), average/mean risk (Chapter 1).

$\rho_i(t_1, t_2)$ — correlation function of output effect $\eta_i(t)$ (Chapter 6).

$\rho(t_1; t_2)$ - the normalized correlation function of multiplicative interference (Chapter 8).

$\sigma(t; \lambda)$ - the reference signal of the optimum system of processing.

$\sigma_\lambda^2, \sigma_\xi^2$ — dispersion of the evaluation/estimate of the parameter $\lambda$, the dispersion of arbitrary random variable $\xi$.

$\Sigma=\|\Sigma_{ij}\|$ — matrix/die of the covariances of measuring errors.

$\tau$ - time lag of signal.

$\varphi$ - initial phase of oscillations/vibrations, the modulus/module of vector $\vec{\varphi}$ (Chapter 7).

$\vec{\varphi}$ - vector of spatial frequency (Chapter 7).

$\Phi$ - Doppler frequency shift.

$\chi(\vartheta)$ - the phase antenna radiation pattern.

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$\psi()$ - ambiguity function.

$\Psi()$, $\Psi()$ - complex ambiguity function and its modular value.

$\dot{\Psi}_{\vartheta}()$, $\dot{\Psi}_{z}()$, $\dot{\Psi}_{\tau}()$ — complex ambiguity functions of elementary signal, periodic sequence of elementary signals and cutting function.

$\vec{\omega}$=t, $\vec{r}$ - generalized space-time coordinate.

$\Omega$ - range of change $\vec{\omega}$, angular rate of scanning (Chapter 5).

$\Omega(f; \vec{\varphi})$ - three-dimensional/space - temporary/time energy interference spectrum.

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